# **Oriental motor**

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Technical Manual Stepper Motor Edition

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# **History of Stepper Motors**

A "motor" is a device that converts electrical energy to mechanical power and is required when you move or stop something. A stepper motor<sup>\*1</sup> is a specific type of motor that offers excellent performance for "moving precisely and stopping accurately." A stepper motor is also usable for high speed rotation or continuous rotation at a constant speed. It can rotate and stop freely from a stop state to a rotation speed of several thousand revolutions per minute. This technical manual explains the structure, rotation principle, and various characteristics so that you can understand and use the hybrid type stepper motors effectively.

Before going into details, let us introduce the history of stepper motors.

It is said that the stepper motor was used in applications starting in the middle of the 19th century, and the variable-reluctance (VR) type stepper motor was used for controlling the direction of the gun barrel on a battleship of the United Kingdom.



VR Type Stepper Motor in Early Stage

Industrial technology in the US was developed significantly by the middle of the 20th century, and AC and DC servo motors became as a driving source for automated machines. A little later than this, with the emergence of numerical control and stepper motor adoption, the characteristics of the VR type stepper motor was improved.

The basic structure of the hybrid type stepper motor consists of a multipole motor with small teeth on a rotor and stator, and a permanent magnet is inserted between rotor cores divided in the axial direction. It was proposed on the patent<sup>\*2</sup> filed in 1952 by the General Electric Company of the US, and there has been no change in the basic structure even today. Using the structure proposed here, SIGMA Corporation of America produced an AC motor for low-speed rotation, which was 72 r/min when AC 60 Hz was applied. The number of poles was 100-pole, and this is also taken over today. This company also produced an electric motor, which obtained a desired rotation amount and rotation speed by switching the current of each phase individually, in the latter 1960s.

\* 1 The stepper motor is also called "pulse motor" because it is controlled using a pulse signal. This technical manual uses "stepper motor" that was often used in USA and was named from step-like movements according to pulses.

\*2 USP2859999, DYNAMOELECTRIC MACHINE

At the end of the 1960s, the hybrid type stepper motor started production in Japan. In the 1970s, due to developments of detection devices such as encoders as well as DC amplifiers, servo motors became mainstream in actuators for numerical control machining tools. On the other hand, stepper motors grew in popularity in output devices of office automation equipment and disk drives.<sup>\*3</sup>

The stepper motor was developed as an AC motor of 2-phase, but it was a problem that the vibration was large if driven by square waves to be rotated freely. If the number of phases is increased, the angle resolution is improved, and the torque fluctuation between steps is made small to decrease the vibration. This causes multiple phases to attempt, and as a result, 2-phase, 3-phase, 4-phase, and 5-phase motors have been introduced. In Oriental Motor, a 5-phase stepper motor has been developed and manufactured under a technical partnership with BERGER LAHR GmbH of the DE.

Attempts to increase the torque by contriving small teeth shapes were performed from an early stage. Figures shown below are the summaries of developments since the 1980s.



Development of Stepper Motor in Recent Years

Taking a look at the change in static torque based on the motor in 1980, the static torque increased 1.5 times around 1986 and doubled around 2000. The "SIGMAX" technology, which was developed by Sigma Industrial Inc of America, was a pioneer in high-torque design. This was a technology to increase the torque by inserting slot magnets in the stator slots (between small teeth). The SIGMAX technology was applied to 5-phase stepper motors by Oriental Motor in 1986, and triggered a competitive environment focused on high-torque hybrid type stepper motors in Japan.

<sup>\* 3</sup> Era of 8-inch floppy disk

A method to increase torque without using slot magnets was studied, and as a result, it was found that the ratio of inner and outer diameters for a stator had a dominant influence on determining the torque. From the late 1980s to the 1990s, the design to increase the ratio of inner and outer diameters was widely developed, and accordingly the static torque was increased approximately 1.5 times higher than before.

From the late 1990s to the 2000s, there was a significant change in the type of magnets used for small motors. Through introducing higher energy magnets and improvements in parts machining technology, the static torque roughly doubled comparing with that around 1980.

A significant change for drive circuits was also developed from the 1980s to 2010. The constant current drive, which used the current feedback control and PWM control (or PAM control), became popular instead of the constant voltage drive. This caused the practical speed to increase. The speed increased from about 60 r/min to 600 r/min around 1985, and to more than 1000 r/min around 2000. Today, it exceeds 1500 r/min.

Although the square-wave drive was popular around 1980 and the step angle was either full step or half step, in the middle of the 1990s, the microstep drive that subdivided the step angle by making the applied current as pseudo-sine waves was adopted. Initially, the microstep drive was the degree to which an electrical angle of 360° was divided into 16. Today, however, due to advances in technology of the current control and PWM resolution, an electrical angle of 360° can be easily divided into thousands of steps.

With developments caused during 30 years since 1980, applications of stepper motors have been expanding. The typical applications in 1980, at which the torque was small and the practical speed was low, were to drive a head for floppy disk drives with high positioning accuracy, or a transmitting-receiving part of a facsimile with high synchronization. In the era from the 1980s to the 1990s, speed and throughput of equipment were increased by the introduction of higher-torque motors and constant current drive circuits. The typical applications at this time were X-Y plotters and full color copying machines. Full color copier machines, which required accurate positioning and low vibration, adopted the 5-phase motors. From the middle of the 1990s, stepper motor applications expanded to moving specimen and rotating spectroscopic mirrors on analytical equipment. It was during this period that stepper motors became widely used in manufacturing equipment for the semiconductor industry. In the 2000s, high-torque and high-speed technologies were advanced further, and low-vibration technology was also advanced due to development in microstep drives. Stepper motor usage were expanded to a variety of automated equipment, robots, 3D printers and other machines.

Developments of the stepper motor with higher torque and lower vibration are still going on, and the stepper motor continues its evolution as a easier-to-use device.

History of Stepper Motors

# **1** Features of Stepper Motors

A stepper motor can rotate at a precise rotation speed and stop at an accurate position without using a position feedback device or a brake. This makes a positioning system possible to configure easily.

## 1.1 Movement of Stepper Motor

A motor is a device to perform continuous rotation, but a stepper motor can achieve precise operation without using a feedback device. It rotates at a specified angle every time a pulse signal is input to a driving circuit (driver).

The stepper motor moves as if continuous rotation is performed when the pulse signal is fast, but it intermittently rotates like the movement of the second hand of the analog clock when the pulse signal is slow. For example, in the case of a 2-phase stepper motor, the basic movement is 1.8° per step as shown in the Fig. 1.1.



Fig. 1.1 2-Phase Stepper Motor

The system to operate the stepper motor consists of three devices. One is a controller or pulse generator which commands the operation by the pulse signal shown in the Fig. 1.2. Another is a driver that supplies electric power to move the motor according to the command. The other is a stepper motor which executes the operation according to the command by converting the electric power to mechanical power.



Fig. 1.2 System for Operation of Stepper Motor

The controller outputs the pulse signal to command the speed and rotation amount of the stepper motor. To perform positioning operation of the stepper motor at high speed, the preset function of the number of pulses and the pulse counter function are required in addition to the acceleration and deceleration control of the pulse signal. Therefore, a dedicated controller for stepper motors or a positioning module of a programmable controller is used for the system.

The pulse signal used for the operation command to the stepper motor is an electric signal generated intermittently like the rhythm of a heart and indicates rectangular waveforms as shown in the Fig. 1.3.





The driver is a device to control the rotation of the stepper motor in accordance with the pulse signal, and consists of a phase sequencer to decide the order of current flow, and a power amplifier to regulate the electric power supplied to the motor.

## 1.2 Positioning with High Resolution and High Accuracy

The stepper motor has a rotation angle that is determined based on the motor structure only, and this rotation angle is called a fundamental step angle.

The driver has a function to subdivide the step angle. And  $\theta_S$ , which is the rotation angle per pulse signal, is determined according to the combination of motor and driver.

The number acquired by dividing one revolution 360° by the step angle is called a resolution.

Resolution = 
$$\frac{360}{\theta_S}$$
 (1.1)

The fundamental step angles are 1.8° for 2-phase stepper motors and 0.72° for 5-phase stepper motors, and the resolutions are 200 and 500, respectively. Some drivers are capable of a subdivision of 100,000 or more, so an extremely high resolution can be achieved. The step angle  $\theta_S$ , which is a rotation angle per pulse, is a theoretically derived value, but the actual rotation angle is infinitesimally different if viewed microscopically.

The typical deviation amount is shown below:

 $\pm 3' = \pm 0.05^{\circ} \text{ or less}$  (1.2)

Errors are not accumulated even if the rotation is repeated, thereby a high accuracy positioning can be performed.

### 1.3 Rotation Amount and Number of Pulses are Proportional

Total rotation amount  $\theta$  of the motor is obtained by the product of the number of pulses *n* input to the driver and the step angle  $\theta_S$ , and represented in the formula (1.3). Therefore, the rotation amount of the motor can be set by setting the number of pulses.

 $\theta = n \cdot \theta_S$ 

As an example, the case for 2-phase stepper motor of the step angle 1.8° is shown in the Fig. 1.4.



Fig. 1.4 Number of Pulses and Rotation Amount

## 1.4 Rotation Speed and Pulse Speed are Proportional

The number of pulses per second is called a pulse speed<sup>\*1</sup>, and the unit is indicated in Hz. If pulses of the frequency 200 Hz are consecutively input to a driver of step angle setting of 1.8°, the motor will rotate one revolution per second (60 r/min). This relationship is shown in the Fig. 1.5.



Fig. 1.5 Pulse Speed and Rotation Speed

The relationship between the pulse speed f [Hz] and the rotation speed N [r/min] is represented as shown in the formula below.

$$N = \frac{\theta_S \cdot f}{360} \times 60 \tag{1.4}$$

## 1.5 Position Holding at Standstill State

Most types of motors are not good at holding at a standstill state, and it is required to use an electromagnetic brake or position feedback to hold the position. On the other hand, a stepper motor can generate a holding force by supplying the motor with a constant flow of current. Therefore, applying an external force to some degree will not cause the stepper motor to rotate. This would be a great advantage for positioning applications.

However, if the motor current is shut off due to power failure or other accidents, the holding force is lost. For safety purposes, in the case of an application where a force is always applied from one direction, such as a vertical drive, a mechanical brake is required to use in combination with the motor.

## 1.6 High Synchronization Performance

During rotation at a constant speed, the motor driving status can be disturbed due to load fluctuations or other factors referred to as disturbance.

Motors, which keep synchronization by using a sensor or other devices, may take time to recover to normal status and largely be displaced from the target position. The stepper motor is a motor with high synchronization performance where the deviation is small from the target position, even with disturbance.

<sup>\*1</sup> A pulse speed is also called a pulse frequency or pulse rate, and pps (pulse per second) that indicates the number of pulses per second is sometimes used as the unit.

1 Features of Stepper Motors

# 2 Stepper Motor Types

There are three types of typical stepper motors available: PM type, VR type, and hybrid type. This technical manual explains with a focus on the hybrid stepper motor, but before that, the PM type and VR type stepper motors are briefly described here.

# 2.1 PM<sup>\*1</sup> Type Stepper Motor

As shown in Fig. 2.1, the PM type is a stepper motor whose rotor surface is made of a permanent magnet. When a current is applied to windings, poles on a stator are magnetized, and magnetic poles on a rotor are fixed at the attracted position.

Fig. 2.1 shows a state in which a current is applied so that the inner circumferential face of  $A_1$  pole is magnetized to the south pole and that of  $A_2$  pole is magnetized to the north pole. The rotor stops at a position where magnetic poles generated in the stator and those on the rotor surface are attracted to each other. If the current is switched from phase A to phase B to magnetize  $B_1$  to the south pole and  $B_2$  to the north pole, the rotor rotates in the clockwise direction by 90°.

A widely used PM type stepper motor is the 2-phase claw pole type<sup>\*2</sup> shown in Fig. 2.2. To achieve a higher resolution, the number of magnetic poles in a rotor is increased, and simultaneously the structure of a stator is changed.

A set of a ring-shaped coil and a case having magnetic permeability<sup>\*3</sup>, which is arranged to surround the coil and whose inner circumferential face is a pole (claw pole) with the same pitch as the magnetic pole on a rotor, constitutes one phase.



Fig. 2.1 PM Type Stepper Motor



Fig. 2.2 Claw Pole Type Stepper Motor

<sup>\*1</sup> Abbreviation for Permanent Magnet

<sup>\*2</sup> A bent pole means a claw.

<sup>\*3</sup> Magnetic permeability means a property that magnetic force is easy to pass through.

Fig. 2.3 shows a cross section including a shaft in a state where a current is applied to phase A. When a current is applied to the coil, a toric magnetic field is generated around the current. In the figure, an upward magnetic field is generated inside. This magnetic field magnetizes the claw pole connected to the lower side of the case to the north pole, and magnetizes the claw pole connected to the upper side to the south pole.

Fig. 2.4 shows the stator as viewed from the inner circumferential side. The upper side and the lower side represent poles of each phase, and the middle N and S arrangements schematically represent the rotor magnetic poles.

In phase A that a current is being applied, the north and south poles are alternately arranged. If the pitch of the stator magnetic poles is the same as that of the rotor magnetic poles, torque is generated between the magnets and the claw poles. Phase A and phase B have the same structure and are offset in a circumferential direction by 1/4 of the pole pitch. If the excitation is switched from the upper stator to the lower stator, the rotor rotates by an amount of the offset.



Fig. 2.3 When Current is Applied to Phase A



Fig. 2.4 Development

The step angle is represented as follows using the number of poles of the rotor  $n_p$  and the number of phases  $n_{\varphi}$ . It can be seen that the number of poles or the number of phases should be increased in order to make the step angle smaller.

$$\theta_s = \frac{360}{2n_p \ n_{\varphi}}$$

To increase the number of poles, the rotor surface must be magnetized at a fine pitch to form a large number of poles (Fig. 2.5). Dozens of poles can be magnetized, but as the number of poles increases, the magnetic force becomes weaker and the torque also becomes smaller. So the number of poles in practical use is up to about twenty some poles.





Fig. 2.5 Multipole Rotor of PM Type Stepper Motor

Claw pole type stepper motors are mostly 2-phase motors. It is possible to make the claw pole type stepper motor multiphase if the number of stators to be stacked is increased, and actually 3-phase motors have also been used.

Although this motor is simple in construction and inexpensive to manufacture, a generated torque is limited because there are limitations on magnets as well as materials of the case that constitutes the magnetic circuit. In addition, the characteristics at high speed rotation is limited due to a large loss during rotation. Therefore, the claw pole type stepper motor is often used for simple positioning.

## 2.2 VR Type Stepper Motor

This is a motor to provide concavity and convexity on the rotor and stator and to rotate by magnetic attractive force between the convexities. Fig. 2.6 shows an example of a 3-phase VR type stepper motor which structure is 8-pole 12-slot.<sup>\*4</sup>

A material that a magnetic force is easily to pass through is called a ferromagnetic material. Typical ferromagnetic elements are iron, cobalt, and nickel. Among them, iron is used as a material for most magnetic cores (iron cores) because its abundance in the earth's crust is large. When an iron core is placed in a magnetic field, a force is applied to pass magnetic flux easily. In this way, the VR type stepper motor generates torque.

Fig. 2.7 shows an example of the VR type stepper motor which structure is the simplest 4-pole 6-slot. Convexities of the rotor are attracted to the magnetized stator poles in a state where current flows so that phase A on the stator upper side will magnetize to the north pole and phase  $\overline{A}$  on the stator lower side will magnetize to the south pole. This is the position where the magnetic flux is most easily to pass through phase A.



Fig. 2.6 VR Type Stepper Motor



Fig. 2.7 VR Type Stepper Motor (4-pole 6-slot)

Fig. 2.8 shows the moment when an exciting phase was switched to phase B.

The lateral convexities of the rotor are partly faced phase B. From this state, a leftward force is generated so that the area facing phase B becomes larger and the magnetic flux can easily pass through, and the rotor rotates by 30° to the position shown in Fig. 2.9. When an exciting phase is switched to phase C, the rotor rotates by 30° to the left as shown in Fig. 2.10 in the same manner.



Following Fig. 2.10, the rotor position changes to the state shown in Fig. 2.7. In this way, the stator pole moves within only a range of 120°, but the rotor rotates in one direction (counterclockwise). This is because a magnet is not used for a rotor of the VR type stepper motor, so the polarity of the stator does not matter, and it is only important which pole is magnetized. Since there is no need to change the polarity of the stator, a current can be flown in one direction. This has an advantage that can simplify a drive circuit.

Rotation of the rotor changes the degree of ease of magnetic flux flowing. This means that inductance of a coil changes depending on the position of the rotor. At this time, the torque T is represented by the following formula, supposing that L is the inductance and i is the current.

$$T = \frac{1}{2} \frac{dL}{d\theta} i^2 \tag{2.2}$$

As can be seen from the formula (2.2), the torque is proportional to the square of the current. To control torque or rotation, it is desirable that the torque is proportional to the current, so the VR type stepper motor is not suitable for fine control. On the other hand, since the rotor is not made by a magnet and the structure is simple and strong, the VR type stepper motor is used as a SRM<sup>\*5</sup> that can rotate at high speed.

The step angle of the VR type stepper motor is represented by the following formula, supposing that  $n_p$  is the number of convexities of a rotor and  $n_{\varphi}$  is the number of phases.

$$\theta_s = \frac{360}{n_p \, n_\varphi} \tag{2.3}$$

<sup>\* 5</sup> Abbreviation for Switched Reluctance Motor. A drive system capable of performing high-speed operation can be created with a relatively simple circuit, so it is attracting attention as a high-speed type power motor.

To subdivide the step angle, it is effective to increase the number of convexities of the rotor. However, if the number of slots of the stator is increased at the same time, the structure becomes complicated, which is difficult to produce. Therefore, "small teeth" that provided a fine concavity and convexity structure on the outer circumference of a rotor and the inner circumference of a stator was devised. Fig. 2.11 shows an example in which 50 small teeth are provided on the rotor and small teeth at the same pitch are also provided on the stator. This is a 4-phase motor which coil is divided into four sets. Substituting  $n_p$  = 50 and  $n_{\varphi}$  = 4 into the formula (2.3) shows that the step angle is 1.8°.



Fig. 2.11 VR Type Stepper Motor with Small Teeth

## 2.3 Hybrid Type Stepper Motor

This is a motor that a magnet is simultaneously used with a VR type stepper motor provided small teeth. It is called as the hybrid type because both the tooth structure and the permanent magnet are provided.

High torque can be obtained by the magnet while high resolution obtained by the small teeth is maintained. Also, the relationship between current and torque is close to proportional, so this motor is easier to control than the VR type stepper motor.

Hereafter, this technical manual focuses on the hybrid type stepper motor.

1 Stepper Motor Types

# **Structure and Principle of Operation** 3 for Hybrid Type Stepper Motor

A common feature of all hybrid type stepper motors is the multipole rotor structure combining "small teeth" and a permanent magnet magnetized in the axial direction. This structure can achieve excellent performance such as high torque and high position accuracy.

#### Structure of Hybrid Type Stepper Motor 3.1

Fig. 3.1 shows the structure of a 2-phase stepper motor, which is the most popular hybrid type stepper motor. It consists of a stator wound with windings, and a rotor inserted with a permanent magnet magnetized in the axial direction. This magnet magnetizes the front and rear rotor segments; one to the north pole and the other to the south pole. The iron cores of the stator and rotor are made by stacking magnetic steel sheets that easily pass the magnetic force and have low loss.

Small teeth<sup>\*1</sup> are provided on the stator and rotor respectively. In the case of the 2-phase stepper motor with a step angle of 1.8°, the number of small teeth on the rotor surface is 50, and the pitch is 7.2°. Two rotor segments are assembled with an offset by a half pitch (3.6°).

Fig. 3.2 shows the rotor viewed from the front side (axial direction). The north pole side is represented in pink and the south pole side is represented in light blue. When viewed the rotor from the front side, the rotor structure contains 100 poles where 50 teeth on the 2 rotor segments are arranged so the south poles (rotor segment 1) line up in between the north poles (rotor segment 2).



Fig. 3.1 Cross-Section Diagram of Stepper Motor



Fig. 3.2 Rotor Viewed from Axial Direction

<sup>\*1</sup> It is also called the "inductor" correctly based on the function to collect and lead the magnetic flux. The term "small teeth" was generalized because the shape looked similar to "teeth" of a gear.

Fig. 3.3 shows a stator core viewed from the axial direction. There are eight main poles<sup>2</sup> on the stator, and small teeth similar to the rotor are provided on the inner circumference. If windings are wound to the main poles and a current is applied, each main pole is magnetized to the north pole or the south pole. To generate a magnetic force by applying current is called excitation. When excited, two poles at the opposing positions have the same polarity, and two poles positioned at 90° from these poles have the opposite polarity. The main poles to be magnetized simultaneously are called "phases" and can be divided into two phases: phase A and phase B. Therefore, this is called a "2-phase stepper motor." The main poles placed at intervals of 45° consist of a group of five small teeth arranged at a 7.2° pitch, and have the same shape.



Fig. 3.3 Stator Core Viewed from Axial Direction

### 3.2 Generation of Holding Force

Fig. 3.4 is an enlarged view of the circled portion in Fig. 3.3 along with the rotor. The small teeth on the main poles of excited phase A become the south pole, and the small teeth on the north pole side of the rotor are attracted. At the same time, the small teeth on the main poles of phase  $\overline{A}$  positioned at 90° become the north pole, and attract the small teeth on the rotor of the south pole side.

A strong attractive force is applied between the rotor and the stator in this state. If an external force is applied to rotate the rotor, a high torque to hold the position is generated against it. This is the reason why the position can be maintained without using an electromagnetic brake.



Fig. 3.4 A State where Phase A is Excited

\*2 The iron core wound the coils and the small teeth on the inner circumferential face are collectively called "main pole."

# 3.3 Teeth Arrange Offset and Step-like Movement

Although the small teeth groups of phase A and those of phase B are positioned at 45°, if 7.2° of the small tooth pitch is considered as a reference, a quarter pitch is offset as shown below.

$$\frac{45}{7.2} = 6\frac{1}{4} \tag{3.1}$$

This offset is called "teeth arrange offset," which has a significant meaning for determining the magnetic structure of the stepper motor.

Fig. 3.5 shows the offset of phase B in a state where phase A is excited to the south pole. When the small teeth groups of phase A are aligned with the small teeth on the rotor, the small teeth groups of phase B are offset from the small teeth on the rotor by 1.8°. This corresponds to the teeth arrange offset.



Fig. 3.5 Offset of Phase B in a State where Phase A is Excited

If excitation is switched from phase A to phase B, the state shown in Fig. 3.6 appears. It shows that the rotor is rotated by 1.8° when the excitation is switched from phase A to phase B. The rotation angle corresponding to this teeth arrange offset is the fundamental step angle of the stepper motor.



Fig. 3.6 A State where Phase B is Excited

At this time, the small teeth are offset by  $1.8^{\circ}$  on the main poles of phase  $\overline{A}$ . If the excitation is switched from phase B to phase  $\overline{A}$ , further rotation of  $1.8^{\circ}$  occurs, and the state shown in Fig. 3.7 appears. Phase A changes to the north pole opposite of the initial state, and the south pole side of the rotor is attracted. A state where phase  $\overline{A}$  is excited to the south pole indicates that phase A is excited to the north pole simultaneously. For this reason, the term phase  $\overline{A}$  refers to the main pole where the small teeth on the stator and those on the rotor are opposite phases. At the same time, it also refers to a state where phase A is excited to the opposite polarity.



Fig. 3.7 A State where Phase A is Excited

In this way, the stepper motor repeats its movement by an angle corresponding to the offset between the small teeth each time the excitation is switched. If the frequency of the pulse signal is increased to a high rate, the intermittent stops between steps are removed, and this causes the movement to be continuous. However, the basic principle of the movement is the same.

### 3.4 Step Angle

#### 3.4.1 Number of Phases and Step Angle

As previously described, the teeth arrange offset of a 2-phase stepper motor is a quarter pitch. Fig. 3.8 shows how the small teeth are offset in a way where the teeth on the north pole side of the rotor are directly aligned with the teeth on the upper and lower main poles. The red characters represent offsets on the north pole sides of the rotor and the blue characters represent those on the south pole sides. The number of states is four because any offset is an integral multiple of a quarter of the teeth arrange offset. If the main poles with a different offset are excited sequentially, they rotate by only the amount of the offset. Therefore, in the case of the 2-phase stepper motor, the angle obtained by dividing the small tooth pitch into four equal parts is the fundamental step angle.



Fig. 3.8 Facing State of Small Teeth of 2-Phase Stepper Motor

Coils are provided in the motor in a state of dividing to several groups. This group of coils is called "phase<sup>\*3</sup>." The coils of the 2-phase stepper motor are divided into two phases, phase A and phase B. A state where current flows in the opposite direction of phase A is called phase  $\overline{A}$ , but a state where the polarity is changed in the same coil is not counted as another phase. The same applies to phase B and phase  $\overline{B}$ . If the direction of current flow is also considered, the number of states of each phase can be doubled. The denominator 4 from the teeth arrange offset is twice the number of phases, and corresponds to the number of different states produced by the current. An offset of the small teeth is determined based on each state, and the excitation phases corresponding to 0, 1/4, 2/4, and 3/4 offsets are A, B,  $\overline{A}$ , and  $\overline{B}$ , respectively. The first step for designing a stepper motor is to evenly arrange the same number of offsets of small teeth as the number of electrical states determined by the number of phases.

The number of states is increased proportionally to the number of phases. The teeth arrange offset can be made finer in inverse proportion to this, and the step angle will also become smaller. There is no theoretical limit for the number of phases. Although it is possible to increase the number of phases in order to obtain finer rotation, the increase in the number of phases causes complications for the motor and the drive circuit, and the output torque also decreases. Therefore, commonly used stepper motors are 2-phase, 3-phase, 4-phase, and 5-phase types at the present moment.

#### 3.4.2 Relationship between Number of Small Teeth and Step Angle

The most popular number of small teeth on the rotor  $Z_R$  in the hybrid type stepper motors is 50, which has not changed since the time when the prototype was proposed more than 60 years ago. The reason why  $Z_R$ =50 continues to be used is not only because it is recognized widely as a standard in stepper motors but also because characteristics such as torque and rotation speed are well balanced. Now, considering the 2-phase stepper motor again, offsets of the small teeth are 0, 1/4, 2/4, and 3/4, which represents the ratio against the small tooth pitch (360/50=7.2° if  $Z_R$ =50). Therefore, if the number of small teeth increases, the small tooth pitch decreases in inverse proportion to it, and the step angle also decreases accordingly. For this purpose, there are motors of  $Z_R$ =100. It is used when a smaller step angle is required.

#### 3.4.3 Determination of Fundamental Step Angle

In summary, the fundamental step angle of the stepper motor is in inverse proportion to the number of small teeth on the rotor  $Z_R$  and the number of phases n. It is obtained by the following formula.

$$\theta_S = \frac{360}{2n Z_R} \tag{3.2}$$

The denominator 2 indicates that the main poles of the stator can be configured into two states, north pole and south pole.

<sup>\*3</sup> In the three-phase alternating current, voltages having a different phase are called phase-U, phase-V, and phase-W, and coils connected them are also called phase-U, phase-V, and phase-W.

### 3.5 5-Phase Stepper Motor

A 5-phase stepper motor has five phases as its name suggests. The number of main poles is generally 10, and the angle between the main poles is 36°. If the number of small teeth on the rotor is  $Z_R$ =50, the small tooth pitch is 7.2°.

$$\frac{36}{7.2} = 5$$
 (3.3)

Since the pitch between the main poles is divisible by the small tooth pitch, when phase A is excited and its teeth are aligned with the teeth on the rotor, the small teeth of other phases are also aligned with other teeth from the rotor. This will not cause rotation even if the excitation is switched. Therefore, the shape of the main poles for the 5-phase stepper motor is intentionally changed to produce the teeth arrange offset.

Fig. 3.9 shows the structure of the 5-phase stepper motor. The number of main poles is 10, and the poles are evenly arranged at intervals of 36°. The number of small teeth on the rotor is 50, which is the same as that of the 2-phase stepper motor, and the small tooth pitch is 7.2°. It can be seen that the shape of the main poles is changed in order to produce the teeth arrange offset. The offset of the small teeth on other main poles when the small teeth on the rotor and stator of phase A are aligned is shown on the outer circumference with the small tooth pitch as a reference. The red characters represent offsets on the north pole side of the rotor and the blue characters represent those on the south pole side.

If the same relationship is compared with Fig. 3.8 described with the 2-phase stepper motor, the difference in the offset between adjacent main poles, which was 1/4 in the 2-phase, is enlarged to 6/10. This is to inverse the magnetic polarity of adjacent main poles.

If offsets of the small teeth are consecutive, the main poles arranged as shown in Fig. 3.10 are excited to the same polarity. This biased pole arrangement results in poor balance and characteristics.



Fig. 3.9 Structure of 5-Phase Stepper Motor



Fig. 3.10 When Adjacent Poles Have the Same Polarity

Actually, as shown in Fig. 3.11, the polarity can be evenly arranged.



Fig. 3.11 Pole Arrangement of 5-Phase Stepper Motor

Fig. 3.12 is an enlarged view showing phase B when the small teeth on the rotor and stator are aligned the north pole side of phase A. Using the small teeth on the stator as a reference, the small teeth on the north pole side are offset by 2.88° clockwise, while the small teeth on the south pole side are offset by 0.72° counterclockwise. Therefore, if this main pole is excited to attract the small teeth on the south pole, a rotation of 0.72° occurs as shown in Fig. 3.13.

In Fig. 3.13, the small teeth on the north pole side of the rotor and the small teeth on the main pole of phase C are offset by 0.72°. When phase C is excited to the south pole next, a rotation of 0.72° occurs. The 5-phase stepper motor repeats this process.



Fig. 3.12 A State where Phase A is Excited



Fig. 3.13 A State where Phase B is Excited.

3 Structure and Principle of Operation for Hybrid Type Stepper Motor

# **4** Characteristics of Stepper Motors

There are various performance characteristics of stepper motors. This chapter explains the meanings of the characteristics that can be observed.

## 4.1 Static Characteristics

These are the characteristics in a standstill state, which are significant features of a stepper motor.

### 4.1.1 Angle – Torque Characteristics

#### a. Relationship between Small Teeth Offset and Generated Torque

As described in Chapter 3 (principle of operation), if a current is applied to windings (coils) of the stator poles to excite the poles, the small teeth on the stator poles and those on the rotor are attracted, and stopped in a state of direct alignment. This is the same state as the compass needle points north to a standstill. In the same way as the direction can be changed by applying an external force to the compass needle, a small amount of rotation can be caused by applying a torque to the shaft of the stepper motor that is in a standstill state. At this time, the characteristics representing the relationship between the applied torque and the rotation angle of the shaft are called the angle – torque characteristics ( $\theta$ -T characteristics), which are the most important characteristics and the positional relationship between the small teeth on the stator and those on the rotor at each point. The polarity (positive or negative) of the torque is positive in the right direction and negative in the left direction.



Fig. 4.1 Angle – Torque Characteristics ( $\theta$ –T Characteristics)

The point indicated in ① is the standstill position when no external force is applied. If an external force is applied to deviate from this position, a resistive force (torque) is generated to return to this position. So, this point is called stable point. Considering the torque generated by the motor as a restoring force, ① is a local minimum of the potential energy<sup>\*1</sup>.

<sup>\*1</sup> A force can be obtained by differentiating the potential energy.

Near the stable point (1), the larger the displacement is, the torque to return to the stable point (1) is increased, and the rotor stops at the point where the external force and the motor torque are balanced (point (2)). If an external force is further applied, the proportional relationship between the displacement and the generated torque is broken, the slope gradually becomes gentle, and the generated torque is maximized at point (3). This generated torque is called the maximum holding torque, which is represented by  $T_H$ .

If a displacement exceeding point ③ is applied, the distance between the small teeth that were originally facing to each other becomes large, and the force to the left direction is decreased. At the same time, the distance to the adjacent small teeth approaches, and the force to the right direction starts acting. As a result, the force to the left direction is further decreased (point ④). Applying further displacement causes to reach point ⑤, and the distance that the small teeth have been originally faced each other is equal to the distance from the adjacent small teeth. In this case, the attractive forces from both sides are balanced, so the generated torque is 0. This balance is an unstable balance because the torque in the left direction is generated in the left side and the torque in the right direction is generated in the left side and the torque in the right direction is generated in the right side with this point as a boundary point. This point is called the unstable point.

Applying further displacement causes the attractive force to the right small teeth to increase (point B). Next, point O at which the maximum torque  $T_H$  is obtained is passed through and the force is decreased (point B) to reach the next stable point D.

The relationship between the displacement from the stable point to the next stable point and the torque is generally represented by a sine wave. It is represented as follows.

$$T = -T_H \sin Z_R \,\theta_m \tag{4.1}$$

In this formula,  $Z_R$  is the number of small teeth on the rotor described before, and  $\theta_m$  is the rotation angle of the rotor.  $\theta$  is often used as a code representing an angle, so these characteristics are abbreviated as " $\theta$ -T characteristics." The formula (4.1) shows that the  $\theta$ -T characteristics are periodic functions with a period of  $360/Z_R$ . This period is represented as  $\tau_R$  using the lower case of the Greek alphabet of T which represents the time period. When the number of small teeth on the rotor is 50,  $\tau_R$ equals to 7.2° ( $\tau_R$ =7.2°). The height of the  $\theta$ -T characteristics is proportional to the current, so the  $\theta$ -T characteristics can also be represented as follows using the proportional constant  $k_T$  and the current *i*.

$$T = -k_T i \sin Z_R \theta_m \tag{4.2}$$

This proportional constant  $k_T$  is called the torque constant.

The formula (4.2) may be represented as follows by changing the expression of the angle.

$$T = -T_H \sin \theta_e \tag{4.3}$$

 $\theta_e$  is a term to represent an angle at which electrical and magnetic phenomena circulate is considered to be 360°, and is called the electrical angle. In contrast,  $\theta_m$  indicated in the formula (4.1) is called the mechanical angle because it is an angle at which an actual "object" rotates and it can be measured mechanically. The electrical angle and the machine angle have the following relationship.

$$\theta_e = Z_R \; \theta_m \tag{4.4}$$

#### b. Expression for Step-like Movement Using heta-T Characteristics

The  $\theta$ -T characteristics are expressed in the same form and in different phases for each phase excited. Fig. 4.2 shows this, and the  $\theta$ -T characteristics corresponding to A, B,  $\overline{A}$ , and  $\overline{B}$  phases of a 2-phase stepper motor are superimposed on each other.

The period  $\tau_R$  is represented in the mechanical angle, but as clearly shown in the figure, it corresponds to 360° in the electrical angle.

The phase difference  $\tau_R/4$  between  $\theta$ –T characteristics corresponds to the offset of the small teeth on adjacent main poles used in the description of the rotation principle, and is 90° in the electrical angle. The stable point for when phase A is excited is at the position of 0 in the figure.



Fig. 4.2  $\theta$ -T Characteristics for Each Phase

If the excitation is changed to phase B, the  $\theta$ -T characteristics (red) of phase A disappears, and the  $\theta$ -T characteristics (blue) of phase B appears. A positive torque, that is, a torque in the right direction is generated at the position of 0 in the  $\theta$ -T characteristics of phase B, so the rotor rotates in the right direction as indicated by the arrow in the figure, and moves to the position of  $T_R/4$ , which is the stable point of phase B. This is the movement of the stepper motor viewed from the  $\theta$ -T characteristics, and the explanation that the step angle is  $T_R/4$ .

#### c. When Multiple Phases are Excited

So far we have described the case to excite one phase, but more often see that multiple phases are simultaneously excited in actual cases. Fig. 4.3 shows a state where the  $\theta$ -T characteristics for when both of phase A and phase B are simultaneously excited are added to the respective  $\theta$ -T characteristics of phase A and phase B at the same current value.

Phase A and phase B operate independently of each other. So, by simply adding their torque, the  $\theta$ -T characteristics for when the two phases are excited can be obtained. The state in which one phase is excited is called 1-phase excitation, and the state in which two phases are excited is called 2-phase excitation.



Fig. 4.3  $\theta$ –T Characteristics When Two Phases are Excited

#### d. Expressing by Vectors

Focusing on amplitude and phase, the  $\theta$ –T characteristics can be represented using vectors as shown in Fig. 4.4.

Considering a plane with the real axis and the imaginary axis<sup>\*2</sup> as shown in the figure, it is assumed that Ta is a vector representing the torque of the phase A, Tb is a vector representing the torque of the phase B, and the angle is fixed at 90° in the electrical angle. If both Ta and Tb rotate in counterclockwise according to the rotation of the rotor and the projection of each vector to the real axis represents torque, Fig. 4.3 and Fig. 4.4 are equivalent. At this time, the torque of 2-phase excitation is represented as the vector sum including the vector length corresponding to the maximum torque and the phases.

Even if the vector rotates by the rotation of the rotor, the relationship between the torque vectors for each phase is fixed. This relationship is represented by omitting the coordinate axis as shown in Fig. 4.5. Eight vectors of A, AB, B,  $\overline{BA}$ ,  $\overline{A}$ ,  $\overline{AB}$ ,  $\overline{B}$ ,  $\overline{B}$ A are shown in this figure. A vector represented by two characters, such as AB, is an abbreviated representation of the vector sum of A+B. Therefore, the magnitude of the torque for each phase and the angle between phases are represented as a relationship between vectors, and this is called the torque vector.



Fig. 4.4 Representation by Torque Vectors



Fig. 4.5 Torque Vectors of 2-Phase Stepper Motor

The angle between vectors represents the phase in the electrical angle. Although the fundamental step angle of the 2-phase stepper motor is 90° in the electrical angle, repeating 1-phase excitation and 2-phase excitation can cause the motor to operate at a step angle of 45°, which is half of the fundamental step angle. In a 2-phase stepper motor with 50 teeth on the rotor, the rotation is 0.9°, which is half of the fundamental step angle of 1.8°. This movement that makes the step angle a half of the fundamental step angle by changing the number of excited phases is called the half-step drive. On the other hand, the movement at the fundamental step angle is called the full-step drive.

<sup>\*2</sup> When a complex number is expressed in a two-dimensional plane, a real number is referred to as a real axis (Re) in the horizontal direction, and an imaginary number is referred to as an imaginary axis (Im) in the vertical direction.

#### e. Measurement Example of $\theta$ -T Characteristics and Detent Torque

Fig. 4.6 shows the  $\theta$ -T characteristics measured by exciting a 2-phase stepper motor with several levels of current values. Torque is basically proportional to current, so curves with different amplitudes are superimposed. The red line represents the characteristics of 2-phase excitation and the blue line represents those of 1-phase excitation. The black line represents a torque generated due to the effect of the rotor magnet even when the current is zero (0), and it is called the detent torque<sup>\*3</sup>.

The maximum torque at 2-phase excitation is the holding torque ( $T_H$ ).

Fig. 4.7 shows the data measuring only the detent torque for one rotation. A constant offset is due to a loss generated in the motor, and mainly due to the hysteresis loss in the iron core. Fine fluctuations are due to slight unbalances in the attractive force of each small tooth.

In detent torque, there are two opposite types of requests. One is for the highest torque, which is preferred for use as a simple brake. The other is for the lowest torque, which is preferred in order to suppress losses. As a reference value, Oriental Motor provides the average value shown by the red line in the figure.

#### f. $\theta$ -T Characteristics of 5-Phase Stepper Motor

The small tooth pitch on the rotor can be divided into 10 for the 5-phase stepper motor, and the  $\theta$ -T characteristics corresponding to each phase are represented in sine waves with a phase difference of 36°, as shown in Fig. 4.8.



Fig. 4.6 Example of  $\theta$ -T Characteristics







Fig. 4.8  $\theta$ -T Characteristics of 5-Phase Stepper Motor (1-Phase Excitation)

<sup>\*3</sup> The detent torque is also commonly generated in motors using a magnet other than the stepper motor. It is called the cogging torque in a brushless motor or a servo motor.

Fig. 4.9 shows the  $\theta$ -T characteristics of 1-phase excitation, 2-phase excitation, 3-phase excitation, 4-phase excitation, and 5-phase excitation.

A 5-phase stepper motor has many phases, and it causes the  $\theta$ -T characteristics diagram to be complicated and difficult to understand. However, if it is represented using the torque vectors as shown in Fig. 4.10, the relationship for each phase and what happens when multiple phases are excited can be intuitively understood.

From Fig. 4.10, as in the case of the 2-phase stepper motor, it can be seen that the step angle of the 5-phase stepper motor when the number of excited phases is the same (ABCDE  $\rightarrow$  BCDEA) is the fundamental step angle, and the half-step drive is obtained if the number of excited phases is changed (ABCD  $\rightarrow$  ABCDE).

The torque vector of each phase is a phase difference of 36°, and the torque vector length  $|T_n|$  for when the phase-n is excited is represented as follows by combining the vectors.

$$T_1$$
 = 1,  $T_2$   $\approx$  1.90,  $T_3$   $\approx$  2.62,  $T_4$   $\approx$  3.08,  $T_5$   $\approx$  3.24



Fig. 4.9 *θ*–T Characteristics When 5-Phase Stepper Motor is Excited with Multiple Phases



Fig. 4.10 Torque Vectors of 5-Phase Stepper Motor

#### 4.1.2 Position Accuracy

The stepper motor can accurately stop in response to a position instruction by a pulse signal, but only a slight difference is generated between the instructed position and the actual stop position. Fig. 4.11 shows that the horizontal axis represents the number of steps and the vertical axis represents the setting rotation angle and the actual rotation angle, using a motor with a step angle of 1.8°. This comparison does not show any difference because the stepper motor has a high stop accuracy.

The difference between the actual rotation angle from the initial position and the setting rotation angle is shown in Fig. 4.12. The horizontal axis represents the number of steps. The result of measuring this deviation is called the position accuracy, and is evaluated by three errors, stop position accuracy, step angle accuracy, and hysteresis angle accuracy.



Fig. 4.12 Error for Each Step (Stop Position Accuracy)

#### a. Stop Position Accuracy

The difference between the actual stop position and the theoretical stop position is measured after rotating by one revolution with an arbitrary point as the origin. This is called the stop position accuracy (Fig. 4.12).

As shown in Fig. 4.12, the entire data of the stop position accuracy is in the positive region. This is because the angle is always measured from the initial position. The initial position may not be a truly ideal position as a stepper motor. If the initial position is deviated from the ideal position in the positive direction, the entire data of the stop position accuracy moves in the negative direction, and if the initial position is deviated in the negative direction, it moves in the positive direction. Therefore, if the initial position and the rotation are arbitrary, there is a possibility that the position will deviate in both the positive and negative directions by the width of the stop position accuracy.

The stop position accuracy is more meaningful for the entire width than the value itself. When the magnitude of the error is indicated, it is indicated with a value of half the width between the maximum value on the plus side and that on the minus side shown by the blue line in the figure. In the example of Fig. 4.12, the stop position accuracy is  $\pm 0.0175^{\circ}$ .

#### b. Step Angle Accuracy

Fig. 4.13 shows the difference between the actual rotation angle and the setting rotation angle for each step, and this difference is called the step angle accuracy. In this example, it is recorded whether each rotation angle is larger or smaller than the step angle of 1.8°.

The step angle accuracy is different between the maximum value in the positive direction and that in the negative direction. To indicate the accuracy, the respective values such as  $+0.339^{\circ}$  and  $-0.339^{\circ}$  are indicated.

#### c. Hysteresis Angle Accuracy

Fig. 4.14 shows the difference of measured standstill positions between the clockwise rotation and the counterclockwise rotation. This result shows that the standstill positions are slightly different depending on the rotation direction even if the position commanded by the pulse is the same. This difference in the standstill positions due to the rotation direction is called the hysteresis angle accuracy, and it is important when high accuracy positioning is performed.

The hysteresis angle accuracy is caused by hysteresis of magnetic steel sheets used for the rotor and stator cores.

#### d. Causes of Error

There are several reasons why the theoretical stop position and the actual stop position are deviated. Among them, the most influential cause is variations in torque for each phase. This is explained using Fig. 4.15, which is an enlarged view of the stop position accuracy (Fig. 4.12). The excited phases are also shown in the figure.

It can be seen that the errors become large or small in a repeated manner each time the excitation is switched. Knowledge of torque vectors make understanding of this concept easier.



Fig. 4.13 Step Angle Accuracy







Fig. 4.15 Stop Position Accuracy (Enlarged)

As an example, Fig. 4.16 shows the torque vectors of a 2-phase stepper motor (Fig. 4.5) in which only the length of the phase A is extended by 20%, and with this change, the directions of the vectors corresponding to 2-phase excitation are changed. If the excitation combination shifts from AB to  $B\overline{A}$ ,  $\overline{AB}$ , and  $\overline{BA}$ , the angle between vectors changes from large to small, then repeats.The stop position accuracy also changes as shown in Fig. 4.15.

Two factors cause the torque for each phase to be different. One is the magnetic force is different between phase A and phase B, and the other is the winding current is different, but the same angular error occurs in both cases.

As shown in Fig. 4.17, the detent torque can also cause the angular error.

Even if the torque generated is an ideal sine wave (thin red line shown in the figure), with the variation in detent torque superimposed (green line), the actual torque is distorted as shown by the thick red line. The same applies to phase B shown in the blue line. It can be seen that the interval between the respective stable points, i.e., the points where the torque is zero (0), is different from that in the case where there is no detent torque (enclosed by black circles). This difference may cause an angular error.



Fig. 4.16 Deviation of Vectors due to Torque Unbalance



Fig. 4.17 Effect of Detent Torque on Angular Accuracy

## 4.2 Transient Characteristics

Behaviors when the stepper motor starts moving, stops from a rotating state, or changes in rotation speed are referred to as transient characteristics. This section introduces some typical movements.

### 4.2.1 Single Step Response

Fig. 4.18 shows a response when one pulse is input at a standstill state, and it is called the single step response. It represents the movement of the step angle explained in the principle of operation is observed on the time axis. The movement of the stepper motor is based on this single step response. After the rotor reaches the target position (1.8° in the figure), it stops at the target position with performing damped oscillation by the moment of inertia and the restoring force of the  $\theta$ -T characteristics. The time period for the envelope shown by the red broken line to fall below ±5% from the step angle is referred to as settling time<sup>\*4</sup>, which is a guide for the stop time of the motor.

Fig. 4.19 is an enlarged view to show a single step response versus time. The time to reach the target position after starting is called the rise time, which is a guideline for starting performance.



Fig. 4.18 Single Step Response



Fig. 4.19 Single Step Response (Enlarged View)

The single step response is approximated by the following formula representing the damped oscillation, supposing that J is the moment of inertia, D is the viscosity coefficient,  $T_H$  is the holding torque, and  $Z_R$  is the number of small teeth on the rotor.

$$J\frac{d^{2}\theta_{m}}{dt^{2}} + D\frac{d\theta_{m}}{dt} + T_{H}\sin Z_{R}\theta_{m} = 0 \qquad (4.5)$$

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<sup>\*4</sup> It represents the time from the end of the movement instruction to the end of the actual movement.
Term 1 is a term of the accelerate torque obtained by multiplying the acceleration by the moment of inertia, term 2 is a term of the viscous torque obtained by multiplying the viscosity coefficient by the rotation speed, and term 3 is a generated torque of the motor that is the  $\theta$ -T characteristics themselves. This formula is a transcendental equation and there is no solution that can be represented by an elementary function. However, it can be approximated by the following formula in a range where the amplitude that the restoring force can be regarded as proportional to the displacement is small.

$$\theta \approx \theta_S \, e^{-\frac{D}{2J}t} \cos\left(\frac{\sqrt{4J \, T_H Z_R - D^2}}{2J} \, t\right) \tag{4.6}$$

 $\theta_S$  is the step angle here. The resonance frequency  $f_{res}$  of the vibration can be obtained by the following formula.

$$f_{res} \approx \frac{1}{2\pi} \sqrt{\frac{T_H Z_R}{J} - \frac{D^2}{4J^2}} \approx \frac{1}{2\pi} \sqrt{\frac{T_H Z_R}{J}}$$
(4.7)

The right side is the resonance frequency with no viscosity term, but because the stepper motor has a large torque and a large number of small teeth, the resonance frequency does not differ significantly even if the viscosity term is not taken into account.

### 4.2.2 Movement at Starting and Responsivity

The stepper motor has an excellent responsivity for starting and stopping, and macroscopically responds instantly to a pulse. However, the movement is microscopically accompanied by a delay.

Fig. 4.20 shows the movements of the rotor for when starting at a certain number of pulse speed from the standstill state. Although the movement is basically superimposed to the step response, the next pulse is input in the middle of the vibration, so the vibration may increase as shown in the green line depending on the timing.

Seeing the state immediately after starting (enclosed by a red circle), you can see that the movements are the same in each case. This is because the movement of the single step response is performed until the second pulse is input. The average slope of each movement represents the each average speed. The slope during the movement of the single step response immediately after starting shows that starting at a higher rotation speed is possible.



Fig. 4.20 Movement at Starting

Fig. 4.21 shows a state of starting at two slightly different speeds. The black line represents the movement of the rotor and the stepped red line represents the position of the stable point. The blue line represents the difference between them, i.e., the delay of the rotor. The solid line represents the speed at which the rotor can start, and the broken line represents the speed at which it cannot start and asynchronism is occurred.

The movement up to about 2 ms is almost the same, but the subsequent movement is guite different, and the branching is the amount of "delay." A delay occurs instantaneously from the start because of rotor inertia. The concept of  $\theta$ -T characteristics is effective even during movement. Near the stable point, a force is generated toward the stable point, and the rotor is accelerated by the force. When the delay is smaller than 3.6° ( $T_R/2$ ), the rotor is accelerated by the torque toward the stable point, but if the delay exceeds 3.6°, it cannot be followed because the torque is reversed to negative and decelerated. This state where the rotor could not follow the command is called "asynchronism," and it is a phenomenon occurring when the acceleration or a load is too large.

The amount of the delay varies not only depending on the pulse speed, but also the friction of the load and the moment of inertia. In either case, asynchronism occurs when the delay exceeds  $3.6^{\circ}$ . In this way, it is important to always be aware of the  $\theta$ -T characteristics in order to understand the movement of the stepper motor.

Fig. 4.22 shows the movement of the solid line in Fig. 4.21 with a wider time span. It can be seen that the vibrational movement caused by the delay at starting gradually decreases.



Fig. 4.21 Starting at High Speed



Fig. 4.22 Movement When Starting at High Speed

### 4.2.3 Movement at Standstill

At the time of standstill, the rotor cannot stop microscopically at the same time as a pulse. Fig. 4.23 shows the movement when the pulse is stopped at t=0. After that, the angle zero (0) is the position of the stable point.

The black line represents the movement when the rotation speed is not so high. After the pulse is stopped and slight overshoots occur, the rotor generates damped oscillation to stop. The blue line represents the movement for when the rotation speed is high. Although overshoots occur up to near 3.6° of an unstable point, they slightly do not reach the unstable point, and returns to the stable point.

The red line represents the movement for when the rotation speed is slightly higher, and overshoots exceed 3.6°. When they exceed the unstable point, the torque is reversed and the rotor is accelerated in the clockwise direction to move to 7.2° of the next stable point and stop. Therefore, stopping from the high rotation speed may exceed the specified position.

### 4.2.4 Effect of Inertial Load

Even if the torque generated by the motor is the same, the acceleration is decreased when an inertial load is applied, and the movement becomes slower. Fig. 4.24 shows the effect of the inertial load by using the single step response. The black line represents an unloaded motor, and the red line represents the same motor with a load that is equivalent to three times the rotor inertia. Since the total inertia was increased four times, the vibration period is twice as long, and the vibration damping time is approximately four times longer.



Fig. 4.23 Movement at Standstill





Fig. 4.25 shows the effect that the inertial load applies on the movements at starting. A state of the motor added with three times the moment of inertia of the rotor is also shown.

The red line represents the instructed position, the black line represents the rotor position, the blue line represents the delay, and the solid line represents a state of the motor single unit and the broken line represents that of the motor added with an inertia. It can be seen that increasing the inertia causes the movement to slow, resulting in asynchronism. In this case, it is required to decrease the speed furthermore.

Fig. 4.26 shows the relationship between a load inertia and a speed (pulse speed) possible to start, and this is called "Inertial Load – Starting Frequency Characteristics." The horizontal axis is represented by the ratio based on the rotor inertia.



Fig. 4.25 Change in Movement at Starting Due to Inertia



Fig. 4.26 Inertial Load – Starting Frequency Characteristics

This characteristics can be approximated by the following formula in a range where the torque of the motor does not change significantly, supposing that  $f_0$  is the starting frequency with no load,  $J_R$  is the moment of inertia of the rotor, and  $J_L$  is the moment of inertia of the load.

$$f = \frac{f_0}{\sqrt{1 + \frac{J_L}{J_R}}}$$

(4.8)

### 4.2.5 Effect of Frictional Load

If a friction load is applied, the movement of the rotor becomes slower. Fig. 4.27 shows the movement.

This example shows a case where a frictional load of about 10% of the holding torque is applied. The frictional load does not affect the frequency of the vibration, but the energy of the vibration is absorbed by the frictional load, causing the vibration damping to increase. The rotor stops at a position where the generated torque and the frictional load are balanced, and this position is deviated from the stable point. Accordingly it can be found that the position accuracy deteriorates due to the frictional load.



Due to Frictional Load

The frictional load reduces the torque available to start the motor. Conversely, the remaining torque, which the torque required for starting is deducted from the generated torque of the motor at a certain rotation speed, can be regarded as the torque allowed as the load. The torque allowed as this load is the pull-in torque.

The torque required for starting can be approximated by the following formula.

$$T_{a} = J \frac{\theta_{S}^{2}}{\tau_{R}} \frac{2\pi}{180} f^{2}$$
(4.9)

In this formula, J is the total moment of inertia to be converted to the motor shaft,  $\theta_S$  is the step angle,  $\tau_R$  is the small tooth pitch on the rotor, and f is the starting frequency. Although approximations are largely contained in this formula, it is not a problem in the sizing of a stepper motor because sufficient margins are generally anticipated in order to avoid instabilities due to transient movements.

The following formula is derived from studies with a lesser degree of approximations.

$$f = \frac{2 \times 180}{\pi \theta_S} \sqrt{\frac{T_M(f)}{Z_R J_R}} \sqrt{1 - \left(\frac{T_L}{T_M(f)}\right)^2} - \frac{T_L}{T_M(f)} \cos^{-1} \frac{T_L}{T_M(f)}$$
(4.10)

In this formula,  $T_M(f)$  is the motor torque at the pulse speed f. This formula contains f on both members and an inverse trigonometric function appears on the right side, so a solution method based on the numerical calculation is required.

The comparison result of the traditional formula (4.9) and the newly proposed formula (4.10) is shown in Fig. 4.28. For reference, the result of the starting simulation using the numerical calculation is also shown. The difference between the proposed formula and the numerical simulation is from the effect of the damping factor of the system, and this indicates that the accuracy of the parameters must be increased if the transient movement is accurately represented.



# 4.3 **Dynamic Characteristics**

Characteristics when the motor is continuously moving are called "dynamic characteristics."

### 4.3.1 Speed – Torque Characteristics

Fig. 4.29 shows a characteristics diagram representing the relationship between the rotation speed and the output torque for stepper motor operation. This is called the speed – torque characteristics. The horizontal axis represents the motor rotation speed or pulse speed, and the vertical axis represents the motor output torque. These characteristics are used for determining whether or not the equipment connected to the motor can be operated, and is the most important feature in selecting a motor.

The speed – torque characteristics are not only determined by the performance of the motor. They are largely affected by the type of driver combined with the motor.



Fig. 4.29 Speed - Torque Characteristics

### a. Maximum Holding Torque ( $T_H$ : Holding Torque)

The maximum holding torque is the maximum torque that the position can be maintained when a torque is externally applied to the motor shaft in the standstill state. If any further torque is applied, the motor rotates. The maximum holding torque is roughly proportional to the current flowing through the windings.

The maximum holding torque for when the rated current is applied to the motor is a value specific to the motor.

### b. Pull-Out Torque Curve

If the load torque is gradually applied to the stepper motor during operation, the motor rotation following the pulse signal cannot be continued when the load torque exceeds the pull-out torque, causing asynchronism to occur and the motor to stop. The torque just before asynchronism is called the pull-out torque at the rotation speed. The pull-out torque curve represents a curve connecting the pull-out torques at the respective rotation speeds, and varies greatly depending on the combination of the motor and the driver.

### c. Maximum Response Frequency ( $f_R$ )

The maximum response frequency is the maximum speed point of the pull-out torque curve. This indicates the highest speed that the motor can rotate in combination with the driver.

### d. Maximum Starting Frequency ( $f_S$ )

The maximum starting frequency is the maximum speed that the motor can follow when a speed without an acceleration time from the standstill state is commanded. In order to rotate the motor at a higher speed than the maximum starting frequency, an operation to accelerate up to the target speed is necessary after causing the motor to start at a speed lower than the maximum starting frequency. This operation is called slow-up.

When the motor is rotating at high speed, an operation to decelerate is also necessary at stopping. This operation is called slow-down.

### e. Pull-In Torque Curve

The maximum starting frequency drops due to a frictional load and an inertial load. The relationship between the frictional load and the starting frequency is called the pull-in torque curve from the meaning that the motor can be pulled into a state of synchronism even with friction load. The pull-in torque curve is considered to illustrate the formula (4.10).

### f. Start-Stop Region

This is the region surrounded by the coordinate axes and the pull-in torque curve. In this region, start and stop operations can be performed without acceleration/deceleration time.

### g. Slew Region

This is the region between the pull-in torque curve and the pull-out torque curve. In this region, start and stop operations cannot be performed, so the slow-up from the start-stop region is required. To rotate the stepper motor at high speed, the slew region must be used.

## 4.3.2 Output Power

The output power represents the amount of energy that can be generated. In the case of a motor, the output power is calculated by the product of torque and rotation speed. It is obtained by the following formula, supposing that the output power is P [W], the torque is T [N·m], and the rotation speed is  $\omega$  [rad/s=s<sup>-1</sup>].

$$P = T\omega \tag{4.11}$$

If the rotation speed is represented by N [r/min], the following formula is obtained.

$$P = \frac{2\pi}{60} T \cdot N \tag{4.12}$$

For applications such as belt conveyors, in many cases, the output power requirement can be found empirically based on the type of load and the size of the mechanical equipment, and the motor can be selected with the output power requirement. On the other hand, in a motor used for positioning, such as the stepper motor, a sizing method based on the magnitude of "torque" more closely related to acceleration/deceleration performance than a steady output value is commonly used. For this reason, torque is listed as the most important characteristic of the stepper motor.

In addition, the torque at low speed is a specific value of the motor, whereas the torque at high speed, which is greatly related to the output power, is largely dependent on the driver (drive circuit). This is one of the reasons why an output power for a stepper motor is not described.

### 4.3.3 Vibration Characteristics

The movement of the motor during rotation is generally observed with speed. Oriental Motor uses a DC tacho generator (3 V output at 1000 r/min) to observe the speed of the stepper motor. Fig. 4.30 shows the output waveform of the DC tacho generator at several speeds.



Fig. 4.30 Observation Waveform by DC Tacho Generator

Even a stepper motor that appears to be rotated at a constant speed can be seen with speed fluctuation if viewed microscopically.



The relationship between voltage fluctuations and speed fluctuations is a proportional relationship as shown in Fig. 4.31.

Fig. 4.31 Relationship between Voltage Fluctuations and Speed Fluctuations

The vibration characteristics are represented in a diagram in which the speed of the motor is plotted on the horizontal axis, and the ripple of the output voltage of the DC tacho generator is plotted on the vertical axis. Fig. 4.32 shows the vibration characteristics when a 2-phase stepper motor is driven in full step.

The vibration (speed fluctuations) occurs because of torque fluctuations. Several causes are described as follows.



Fig. 4.32 Vibration Characteristics (2-Phase Stepper Motor)

The large vibrations at low speed are caused by torque fluctuations from step to step. Switching the excitation causes the motor to quickly jump to the next  $\theta$ -T curve as shown in Fig. 4.33. As a result, a speed fluctuation occurs.

Torque fluctuations and vibrations decrease as the ratio of the step angle to the period of the  $\theta$ -T characteristics decreases. Fig. 4.34 shows an example for a 5-phase stepper motor with a smaller step angle.



Fig. 4.33 Torque Fluctuations at the Time of Excitation Switching (2-Phase)

As shown in Fig. 4.35, the vibration can also be reduced by using the microstep drive technology that subdivides the step angle by slightly changing the current by a circuit.

The torque is proportional to the current flowing through the windings, so fluctuating the current may cause the torque to fluctuate, resulting in vibration. For example, if the rotation speed of the motor is fluctuated due to load fluctuations, the current is varied. At this time, the driver changes the output voltage to restore the current, but if this operating state is excessive or the control speed is slow, the speed variation of the motor may increase vibration.



Fig. 4.34 Torque Fluctuations at the Time of Excitation Switching (5-Phase)





### 4.3.4 Synchronizing Stability

The load torgue and the generated torgue are balanced at the operating point in a state where the motor rotates at a constant speed with a load applied. The stability comparison between a stepper motor and a general synchronous motor<sup>\*5</sup> is shown in Fig. 4.36. In the figure, p is the number of pole pairs  $*^{6}$ , and p=50 is applied to the stepper motor with 50 small teeth. The synchronous motor can have various number of pole pairs, but p=5 is applied in Fig. 4.36. Synchronous motors (other than the stepper motors) are operated while changing the magnitude of torque in order to operate as commanded, so the stability of the system is highly dependent on the performance of the driver. In Fig. 4.36, the stability of the motor itself is compared based on the following conditions: The magnitude of the peak torque for both motors is the same, half the peak torque is applied to both motors as the load torque, and the operating point is set to the position of 0°.





The thin solid line represents the load torque, the red line represents the torque of the stepper motor, and the blue line represents the torque of the brushless motor. The operating point represents a point where the generated torque and the load are balanced. If the load is constant, there is no difference between the stepper motor and the brushless motor. The broken line represents  $\pm 20\%$  of the load torque (10% of the peak torque). The torque slope is larger if the larger number of poles are provided, so the position fluctuations become smaller to the load fluctuations. In this case, the fluctuation range of the position for the brushless motor is about 10 times larger than that for the stepper motor.

<sup>\*5</sup> A motor in which magnetic poles do not move on the rotor. The stepper motor is a type of synchronous motors. The opposite concept is an induction motor.

<sup>\*6</sup> Number of pairs of north and south poles on rotor.

Fig. 4.37 is the same graph enlarged in the horizontal direction in order to show this relationship better.

The brushless motor is large in the variation width of the position and speed in the event of the load fluctuation, and it may lose synchronism. In order to avoid this, it is necessary to monitor the rotor position using an encoder, Hall IC, or the like. On the other hand, the stepper motor is small in the fluctuation range to torque fluctuations, and is less likely to lose synchronism. In many cases, it can be operated without detecting the rotor position. This is the reason why a stepper motor can be operated in an open loop system.

The magnetic poles of the rotor for a stepper motor are created with small teeth. Therefore, a larger number of poles can be provided compared to other motors. The number of poles are 100 when the number of small teeth is 50, and 200 when it is 100. Other types of motors rarely have more than 10 poles, and this causes a difference in synchronization.



Fig. 4.37 Load Fluctuation and Fluctuation Range of Position

## 4.4 Temperature Characteristics

### 4.4.1 Heat Generation of Stepper Motor

A motor is a device that converts electrical energy to mechanical energy. The conversion efficiency is not 100% because a loss is generated by the conversion. In the case of small motors, the loss is mainly generated in the windings (coils) and the iron core.

The windings are mostly made of copper wires, so the loss generated in the windings is called the "copper loss." Copper loss (Pc) means Joule loss generated by passing a current through the winding resistance. It is calculated by the following formula, supposing that n is the number of phases, i is the phase current effective value, and R is the phase resistance.

$$P_c = ni^2 R \tag{4.13}$$

In devices that apply the magnetic force such as motors and transformers, the iron cores are used so that the magnetic force efficiently passes through. The iron core contains magnetic steel sheets that were designed to decrease loss, but the loss is still generated when the magnitude and direction of the magnetic forces change inside.

The loss generated in the iron core is called the "iron loss." Iron loss (Pi) is divided into hysteresis loss (Ph) and eddy current loss (Pe).

Hysteresis loss is energy lost when the direction of the magnetic force is changed in the iron core. Its magnitude is proportional to the magnetic flux density (strength of magnetic force) to the power of 1.5 to 2 and is proportional to the speed (frequency). Eddy current loss is the Joule loss generated by the current induced in the iron core when the magnetic force changes. It is approximately proportional to the square of the magnetic flux density and speed (frequency). This current is called the eddy current because it is generated in a circular loop shape around the magnetic flux lines.

### 4.4.2 Rotation Speed and Temperature

The rotation speed – loss characteristics in Fig. 4.38 show the relationship between the motor speed and the loss in a constant current driver.

At low speeds, the frequency of the magnetic force change is low, and the loss is mostly copper loss. The total loss is increased as the speed is increased. This is because the iron loss is increased. In particular, the eddy current loss is proportional to the square of the frequency to cause a steep change. It is believed that the increase in total loss, while both the iron loss and the copper loss are decreased, at high speed is based on the increase in the mechanical loss due to resistances of ball bearings or air.

The peak of the loss appears near the center of the figure, and the loss is decreased at the high speed side. The loss is decreased as the frequency is increased. This is because the current and the magnetic flux density are decreased. Fig. 4.39 shows the voltage applying to the motor, the current flowing in the motor, and the magnetic flux density of the iron core under the same conditions as in Fig. 4.38.



Fig. 4.38 Rotation Speed – Loss Characteristics



Fig. 4.39 Change in Speed and State

Impedance is the resistance of the motor windings as viewed from the driver. The induced voltage generated from the motor is approximately proportional to the speed. In order to flow a constant current against this, it is necessary to increase the voltage incrementally. This is region A in the figure, and it can be seen that the voltage rises in this area from the low speed to the vicinity of the 680 r/min. The magnetic flux density is not changed substantially, and the iron loss is increased due to the effect of frequency, and the total loss is also increased. This speed region is called the "constant current region" because the current is kept at a constant level.

In region B, the voltage is the maximum value that can be output. This causes the current to be lower against the speed, and the copper loss is decreased. In this region, the magnetic force generated by the current acts to suppress the magnetic force generated by the magnet, and the magnetic flux density is also decreased. For this reason, the iron loss is decreased although the frequency becomes higher. It is called the "constant voltage region" because the voltage is at a constant level in this area.

The boundary between the constant current region and the constant voltage region is located at a low speed side. Semilogarithmic charts<sup>\*7</sup> are used from Fig. 4.38 to Fig. 4.40 so that the change can be easily confirmed.

The loss causes heat and increases the motor temperature. At the speed of a large loss, the motor temperature also becomes high. Fig. 4.40 compares the data of the motor surface temperature rise for when continuous operation is performed (with heat sink attached) to the loss data of Fig. 4.38. Fig. 4.41 plots the same data as the relationship between the loss and the temperature rise, and shows that both are approximately in a proportional relationship.







Fig. 4.41 Relationship between Loss and Temperature Rise

\*7 A graph in which one axis is logarithmic. The low speed side is widely shown and the high speed side is narrowly shown.

#### 4.4.3 **Operating Duty and Temperature**

The stepper motor is mostly used for positioning purposes, and is not generally used for continuous rotation. Intermittent operation, which repeats rotating and stopping, is mainly used. In this case, the average loss  $P_{ave}$  is represented as follows, supposing that D is the operating duty,  $P_{run}$  is the loss during operation, and  $P_{stop}$  is the loss at standstill.

(4.14)

100

80

64

$$P_{ave} = P_{run} \times D + P_{stop} \times (1 - D)$$

The average loss varies linearly to the operating duty, and the temperature becomes linear to the approximate loss, so it can be assumed that the motor temperature also varies linearly to the operating duty. In the example shown in Fig. 4.42, the temperature rise at 100% of the operating duty (continuous operation) is 100 °C, and the temperature rise at standstill is 40 °C. If the linear relationship between the operating duty and the temperature rise is used, it can be seen that an operating duty of about 67% is needed when the temperature rise is 80 °C, and that the temperature rise when the operating duty is 40% is about 64 °C.

As shown in Fig. 4.41, the temperature and the loss are not strictly a linear relation, and the relationship between the operating duty and the temperature is also not strict.

### Noise Characteristics 4.5

#### 4.5.1 **Basic Knowledge about Sounds**

[emperature Rise [°C] 40 0 0 40 67 100 Operating Duty [%] Fig. 4.42 Relationship between Operating Duty and Temperature

Sounds are pressure fluctuations in air captured by an eardrum. This pressure fluctuation is called the sound pressure and is represented by pascal Pa=N/m<sup>2</sup>, which is a unit of pressure. The range of the sound pressure that people can capture is said to be  $2 \times 10^{-5} \sim 2 \times 10^{7}$  Pa. It is difficult to show the range as large as 10<sup>12</sup> on a linear scale, and human senses respond logarithmically to the stimulus intensity. Therefore, bell [B] that is the common logarithm of the sound pressure ratio with 2×10<sup>-5</sup> Pa of the minimum audible pressure, and decibel [dB] that is obtained by dividing the bell by 10 are called the sound pressure level. They are used as a unit of the magnitude of sound. Using this unit, the audible range of people in the sound pressure direction is 0 to 120 dB.

The audible range in the frequency direction is 20 Hz to 20 kHz. The audibility of sound in this range varies depending on the frequency, as shown in Fig. 4.43. Fig. 4.43 shows how the magnitude of sound that people perceives varies with frequency. For example, the 200 Hz sound is 10 dB smaller than the actual sound pressure level. Conversely, the sound near the 2 kHz is larger than the actual sound pressure. The result of performing the correction on the measured sound pressure level shown in Fig. 4.43 is the A-weighted sound pressure level dB(A), which is also called the noise level.



### 4.5.2 Sounds by Motor

There are several types of sounds generated by the motor; vibration sound due to vibration of the shaft, magnetic sound due to deformity by a magnetic force, and bearing noise generated from bearings. Among them, the vibration sound and the magnetic sound are considered as the main noises for a stepper motor.

The vibration sound is not generated by the motor itself, but is generated by torque fluctuations transmitted to the equipment through the shaft as an excitation force. It increases at the speeds where vibration of the motor is the highest, such as low speed or resonance frequency. Depending on the frequency of the vibration, the entire equipment may resonate to amplify the noise level. The rotor and the stator are magnetically attracted to each other in the motor. Therefore, if the rotor vibrates, the stator also vibrates due to its reaction. This vibration is transmitted from the motor mounting surface to the equipment. If the motor mounting surface is low in strength, the vibration may be transmitted to the entire equipment, causing noise to generate.

The magnetic sound is often noticed as the sound generated by the motor. The magnetic sound is caused by a minute deformity of the motor due to attractive forces and magnetostriction of the rotor and stator. The deformity amount is in the order of 1  $\mu$ m, so it cannot be seen visually and is hardly felt by touch. However, the noise level for when the displacement is the same is proportional to the square of the frequency. If a motor with a high number of poles is driven at a high frequency, dissonant noises are generated.

Fig. 4.44 shows the results of measuring the relationship between the rotation speed and the noise level. The vertical axis represents the sound power level that indicates the average noise level at 1 m distances. Overall, there is an upward trend relative to the frequency, but this shows the effect that is increased with the square of the frequency. Some peaks are the speeds at which the excitation force frequency matches the natural frequency for the deformity of the motor.



Fig. 4.44 Change of Noise Level at Rotation Speed

Fig. 4.45 shows the spectrum of the magnetic sound when a 2-phase stepper motor is rotated at the speed of 4 kHz with the full-step drive method. The frequency of the spectrum is composed of the frequency of an integral multiple of 1 kHz. The fundamental tone 1 kHz is the frequency of the current inside the motor at this speed, and is also the frequency of the change of the magnetic flux in the iron core.

The magnetic noise is radiated directly from the surface of the motor, but may be transmitted from the motor mounting surface to the equipment because of a slight surface vibration. Fig. 4.46 shows the results of measuring the vibration of the motor mounting surface using an accelerometer. It shows a high acceleration is generated. If this vibration is transmitted from the mounting surface to the equipment, it may cause a high noise level.





Fig. 4.46 Acceleration of Mounting Surface

4 Characteristics of Stepper Motors

# 5 Drive Circuit

When a motor is used for position control or speed control, a drive circuit (driver) is necessary. This chapter explains basic features of the driver for a stepper motor and a new driving method.

# 5.1 Wiring Method

Multiple windings are provided inside the motor. They are internally segmented, and lead wires are fed out externally from specific points of the windings. An understanding of internal connections is significant because it is also related to the type of driver that can be combined with the motor.

### 5.1.1 Bipolar Drive and Unipolar Drive

To switch the stator poles between north pole and south pole, there is a method to change the direction of the current flow through the windings in order to energize the windings for north pole and south pole separately. The former is called the "bipolar drive" because of its ability to send current in both directions. The latter is called the "unipolar drive" because it can only send current in one direction. The basic circuit of the bipolar drive system is shown in Fig. 5.1, and that of the unipolar drive system is shown in Fig. 5.2.



Fig. 5.1 Bipolar Drive Circuit



The bipolar drive system is a driving method to provide a full-bridge circuit for each phase. When transistors Tr1 and Tr4 are turned ON, the current flows in the direction of the solid line, and when transistors Tr2 and Tr3 are turned ON, the current flows in the direction of the broken line. Four transistors are necessary for one phase of the motor windings, so this was considered an expensive driver circuit design during the early stages of stepper motors when electronic components were more expensive.

The unipolar drive system is a driving method in which only one transistor is turned ON to switch the windings. A circuit where only two transistors are necessary for one phase of the motor windings is an advantage of simplifying the output circuit. Therefore, this circuit was well received in the past. However, there is a disadvantage since only half of the windings can be used.

As shown in the formula (5.1), the motor torque  $T_M$  is proportional to the product of the current i and the number of turns t of the windings.

$$T_M \propto i \times t$$
 (5.1)

Copper loss is proportional to the square of the current and the resistance.

 $P_c \propto i^2 R$ 

(5.2)

If the diameter of the copper wire used as the windings is constant, the resistance is proportional to the number of turns of the windings. In other words, this relationship is proportional to the utilization ratio of the windings. The utilization ratio of windings k leads the following.

$$\begin{cases} R \propto k \\ t \propto k \end{cases}$$
(5.3)

And the above relationship can be summarized.

$$T_M \propto \sqrt{k}$$
 (5.4)

It can be seen that the torque of the stepper motor is proportional to the square root of the utilization rate of windings. The utilization ratio of windings for the unipolar drive system is half of the bipolar drive system, and the torque can be as low as about 70%. As a result, the unipolar drive system decreased in popularity as the price of electronic components decreased, and the proportion of the bipolar drive systems has increased in the market.

### 5.1.2 Drive System of a 2-Phase Stepper Motor

There are four types of 2-phase stepper motors with four, five, six, and eight lead wires. Fig. 5.3 shows each internal connection. Colors described in terminals are the standard colors of lead wires used in our stepper motors.



Fig. 5.3 Internal Connections of 2-Phase Stepper Motor

The motor of four lead wires is designed to work with the bipolar drive.

The motor of five lead wires is designed to work with the unipolar drive.

The motor of six lead wires is designed to work with the unipolar drive. However, this motor can be used in the bipolar dive by insulating the center taps (yellow terminal on phase A side and white terminal on phase B side) and using terminals at both ends of the coil.

The motor of eight lead wires can be used for three types of driving methods. One is the unipolar drive in which one of the windings is connected to the power supply (Fig. 5.4), another is the bipolar series drive in which the windings are connected in series (Fig. 5.5), and the other is the bipolar parallel drive in which the windings are used in parallel (Fig. 5.6).





Fig. 5.5 Bipolar Series Drive



Fig. 5.6 Bipolar Parallel Drive

The unipolar drive and the bipolar drive have different torque magnitudes because of the different utilization ratios of the windings. In addition, the bipolar series drive method has an impedance of four times with respect to the bipolar parallel drive method, so the high-speed torque drops significantly. Fig. 5.7 shows the torque characteristics when these three types of connections are compared.





### 5.1.3 Drive System of a 5-Phase Stepper Motor

The most basic drive method for a 5-phase stepper motor is the standard bipolar drive system shown in Fig. 5.8. This method, which provides the full bridge in each phase, is superior in terms of characteristics, but is expensive because of the large number of power devices.



Fig. 5.8 Standard Bipolar Drive System

Three more types of drive systems, which are the star bipolar drive system, the pentagon drive system, and the new pentagon drive system, have been devised. These are drive systems where phase windings are connected to each other, and the number of connections with the driver is reduced. Output sections of these drivers consist of five half bridges, and the number of power devices is half that of the standard bipolar drive system. There is no difference in characteristics between them.

Fig. 5.9 is a connection diagram of the star bipolar drive system. This is a drive method where one side of each phase is connected to a neutral point.



Fig. 5.9 Star Bipolar Drive System

Fig. 5.10 is a connection diagram of the pentagon drive system, and Fig. 5.11 is that of the new pentagon drive system. In both drive systems, the windings are annularly connected, and motor lead wires are connected to the driver. The difference is the order in which the phase windings are connected. Oriental Motor does not have a motor for the pentagon drive, so the connection method using a motor of 10 lead wires for the standard bipolar drive is shown here.



Fig. 5.10 Pentagon Drive System



Fig. 5.11 New Pentagon Drive System

# 5.2 Excitation Mode and Excitation Sequence

A group of windings that switches current at the same time is called a phase. There are eight windings in the stator of a 2-phase stepper motor, and these windings are divided by four into two groups: phases A and phase B. This is the reason why it is called 2-phase. A 5-phase stepper motor has ten windings, and these windings are divided by two into five groups: phase A, phase B, phase C, phase D, and phase E. The number of windings can be increased by changing the structure of the motor, but if they are divided into two groups, the motor is a 2-phase motor, and if they are divided into five groups, the motor is a 5-phase motor.

Excitation mode determines how many phases of a stepper motor are simultaneously excited. "Excitation sequence" determines the correct order of phases to apply current to.

### 5.2.1 Excitation Mode and Excitation Sequence of 2-Phase Stepper Motors

The torque vector of a 2-phase stepper motor is shown in Fig. 5.12 again. Considering the current direction and combination, the eight torque vectors shown here are all possible combinations for the 2-phase stepper motor.

The 1-phase excitation mode is a method only one phase is always excited, and only the torque vectors of the vertical and horizontal 1-phase excitation are used. The corresponding excitation sequence in the counterclockwise rotation is as follows.



Fig. 5.12 Torque Vectors of 2-Phase Stepper Motor

Counterclockwise rotation:  $A \rightarrow B \rightarrow \overline{A} \rightarrow \overline{B} \rightarrow$ 

The excitation sequence is returned from the end to the top to repeat. In the case of the clockwise rotation, the excitation sequence is as follows.

Clockwise rotation:  $\overline{B} \rightarrow \overline{A} \rightarrow B \rightarrow A \rightarrow$ 

The rotor rotates by one small tooth when the excitation sequence completes an electrical cycle. The rotation angle for each step is a quarter of the small tooth pitch, and 1.8° for a stepper motor with a 50 small teeth rotor.

The 2-phase excitation mode is a method where two phases are always excited, and only the torque vectors of the diagonal positions are used. The corresponding excitation sequence is as follows.

Counterclockwise rotation:  $A B \rightarrow B \overline{A} \rightarrow \overline{A} \overline{B} \rightarrow \overline{B} A \rightarrow$ 

Clockwise rotation:  $\overline{B} A \rightarrow \overline{A} \overline{B} \rightarrow B \overline{A} \rightarrow A B \rightarrow$ 

The step angle is 1.8°, which is the same as the 1-phase excitation mode. The length of the torque vector is approximately proportional to the magnitude of the torque, so the torque of the 2-phase excitation is approximately  $\sqrt{2}$  times as that of the 1-phase excitation. However, this is the case when the current applied to the winding is always at a constant state. If the current is the same, the 2-phase excitation generates in twice as much copper loss as the 1-phase excitation. If the copper loss is the same, the current at the time of the 1-phase excitation is  $\sqrt{2}$  times, so the torque becomes the same. When the number of excitation phases changes, precaution must be taken to determine whether the current in each phase is constant or the copper loss of the entire motor is constant.

The 1-2 phase excitation mode, which repeats 1-phase excitation and 2-phase excitation, is a method to use all torque vectors. The corresponding excitation sequence is as follows.

In this case, the excitation sequence completes an electrical cycle in 8 steps, so the step angle is 0.9°. This method changes the number of excitation phases, which makes the step angle half of the fundamental step angle.

The sequence of 1-phase excitation is shown in Fig. 5.13 instead of describing the codes in series. The order of the excitation sequence is represented on the horizontal axis, and the positive and negative states of the phase currents are represented (up and down) from the baseline. In this figure, the codes of the excited phases are also shown, but only the figure is usually shown. Reading this figure from left to right results in the excitation sequence in the counterclockwise rotation, and reading from right to left results in the excitation sequence in the clockwise rotation. This shows the sequence and direction of the excitation phases.



Fig. 5.13 1-Phase Excitation Sequence

Similarly, Fig. 5.14 shows the sequence of 2-phase excitation and Fig. 5.15 shows the sequence of 1-2-phase excitation.



Fig. 5.14 2-Phase Excitation Sequence



Fig. 5.15 1-2 Phase Excitation Sequence

### 5.2.2 Excitation Mode and Excitation Sequence of 5-Phase Stepper Motor

A 5-phase stepper motor can perform 1-phase, 2-phase, 3-phase, 4-phase, and 5-phase excitation modes, and combined excitation modes. However, the 1-phase, 2-phase, and 3-phase excitation modes have small torque, so three types of excitation modes, which are the 4-phase excitation mode, 5-phase excitation mode, are used.

Fig. 5.16 shows the torque vectors for 4-phase excitation (broken line) and 5-phase excitation (solid line).



Fig. 5.16 Torque Vectors of 5-Phase Stepper Motor

The excitation sequence of the 4-phase excitation mode where four phases are excited is as follows. A B C D $\rightarrow$ B C D E $\rightarrow$ C D E  $\overline{A} \rightarrow$ D E  $\overline{A} \ \overline{B} \rightarrow$ E  $\overline{A} \ \overline{B} \ \overline{C} \rightarrow \overline{A} \ \overline{B} \ \overline{C} \ \overline{D} \rightarrow \overline{B} \ \overline{C} \ \overline{D} \ \overline{E} \rightarrow \overline{C} \ \overline{D} \ \overline{E} \ A \rightarrow \overline{D} \ \overline{E} \ A \ B \rightarrow \overline{E} \ A \ B \ C \rightarrow \overline{A} \ B \ \overline{C} \ \overline{D} \rightarrow \overline{B} \ \overline{C} \ \overline{D} \ \overline{E} \rightarrow \overline{C} \ \overline{D} \ \overline{E} \ A \rightarrow \overline{D} \ \overline{E} \ A \ B \rightarrow \overline{E} \ A \ B \ C \rightarrow \overline{C} \ \overline{D} \ \overline{E} \ A \rightarrow \overline{D} \ \overline{E} \ A \ B \rightarrow \overline{E} \ A \ B \ C \rightarrow \overline{C} \ \overline{D} \ \overline{E} \ A \rightarrow \overline{D} \ \overline{E} \ A \ B \ \overline{C} \ \overline{D} \ \overline{C} \ \overline{C} \ \overline{D} \ \overline{C} \$ 



Fig. 5.17 4-Phase Excitation Sequence



Similarly, the 5-phase excitation sequence is shown in Fig. 5.18, and the 4-5-phase excitation sequence is shown in Fig. 5.19.

# 5.3 Drive System

Whereas the wiring method and the excitation method determine the combination and the order in which current flows through the windings of the respective phases, the method for determining the magnitude of the current is the driving system.

### 5.3.1 Constant Voltage Drive System

The constant voltage drive system is a method to drive the motor in a state where the output voltage of the drive circuit is kept constant. The configuration of the drive circuit is simple, but there is an issue where the output torque decreases at high speed.

The formula (5.5) represents the relationship between voltage V applied to the motor, current I, impedance Z, and induced voltage E of the motor, and is called the voltage equation. The letters in bold indicate that each variable is a vector or a matrix corresponding to the number of phases.

$$\mathbf{V} = \mathbf{Z}\mathbf{I} + \mathbf{E} \tag{5.5}$$

Solving this for the current results in the following formula.

$$\mathbf{I} = \mathbf{Z}^{-1} \left( \mathbf{V} - \mathbf{E} \right) \tag{5.6}$$

The back EMF E is proportional to the speed. The impedance Z changes according to the following formula, supposing that R is the winding resistance, f is the frequency of the current, and L is the inductance.

$$Z = \sqrt{R^2 + (2\pi f L)^2}$$
(5.7)

Both of them increase relative to the speed, so the current decreases relative to the speed if a constant voltage is applied, and the torque also decreases accordingly.

In order to increase the output torque at high speed rotation, by connecting resistors in series with the windings as shown in Fig. 5.20, the impedance is increased.

In this method, the applied voltage Vcc is increased by the added resistor to ensure that current does not decrease. The impedance in this state is as follows.



Fig. 5.20 External Resistor Additional Drive

$$Z = \sqrt{(R + R_0)^2 + (2\pi f L)^2}$$
(5.8)

If compared with the formula (5.7), the effect of frequency is reduced. Therefore, the current reduction at high speed is suppressed.

Although the torque at high speed increases as the external resistor and the applied voltage are increased, loss due to the external resistor increases at the same time, resulting in inefficiency.

### 5.3.2 Constant Current Drive System

In the constant voltage drive, the current decreases corresponding to the increase of the speed in principle. To avoid this problem, the constant current drive system, which cause the current to be constant by changing the voltage, is used.

Fig. 5.21 shows the relationship between the speed, phase current, phase voltage, and torque in a constant current driver.

It can be seen that the voltage is increased corresponding to the increase of the speed, but the current is kept at a constant level. In this example, the upper limit of the applied voltage is about 140 V, and the maximum voltage that can be applied is output at 800 r/min or higher speed. The current decreases corresponding to the increase of the speed because the constant voltage drive is performed in this region.

Fig. 5.22 shows a basic circuit for the constant current drive.

It's recommended to provide a voltage sufficiently higher than the rated voltage of the motor. To ensure motor current is constant, apply feedback by using a current sensing resistor, and adjust the voltage by Pulse Width Modulation (PWM) method. Higher voltage is applied as the speed increases, so higher torque can be obtained.



Fig. 5.21 Current and Voltage at Constant Current Drive



Fig. 5.22 Constant Current Drive Circuit

# 5.4 Microstep Drive

The fundamental step angle is based on the stepper motor structure. If a finer step angle was required, it was necessary to replace to a different motor that has a higher number of phases on the stator or a higher number of small teeth on the rotor. To solve this issue, an idea of the microstep drive, which electrically subdivides the fundamental step angle, was devised.

Using a 2-phase stepper motor as an example, the driving method for the microstep drive is described. Fig. 5.23 shows the current waveform of the half-step drive using the microstep drive, and Fig. 5.24 shows the trajectory of the vertex of each torque vector. In the half-step drive by the 1-2 phase excitation mode described previously, the current in the windings is constant because the excitation direction is changed depending on existence or non-existence of the current. In the microstep drive, however, the step angle is subdivided by providing an intermediate level at the current switching point. The square of the distance from the origin to the vertex of Fig. 5.24 is proportional to the copper loss in the windings, so the step angle can be subdivided under certain loss conditions.



Fig. 5.23 Current at 2-Phase Microstep Drive



Fig. 5.24 Current Trajectory at 2-Phase Microstep Drive

Fig. 5.25 shows the current values of 1/4 step and 1/8 step, and Fig. 5.26 shows the trajectory of the vertex of each torque vector. Increasing the number of intermediate levels of the current causes the step angle to be smaller, and the shape of the entire waveform to be closer to a sine wave.



Fig. 5.26 Current Trajectory at Microstep Drive

In the case of an arbitrary step angle, the length of the torque vector  $i_0$  is determined first, then a desired step angle  $\theta_m$  is determined. At this time, the phase current is switched so that the phase current of the *n*-th step is a sine wave with a phase of 90° as follows.

$$\begin{cases} i_A = i_0 \cos(nZ_R \ \theta_m) \\ i_B = i_0 \sin(nZ_R \ \theta_m) \end{cases}$$
(5.9)

In the microstep drive of a 2-phase stepper motor, since the current waveform approaches a sine wave, the excitation sequence seems to be quite different from the normal excitation sequence (i.e. full step and half step), but it is actually an extension of the excitation sequence. In regards to this, an example of the 5-phase microstep drive is shown. Fig. 5.27 shows the current when the full-step drive is divided into 10 divisions using the 5-phase microstep drive. As compared with the fullstep drive, only the portion where the current is switched is changed, and it can be seen that the microstep drive is an extension of the excitation sequence (Fig. 5.19).



Fig. 5.27 Current at 5-Phase Microstep Drive

# 5.5 New Driving Method

## 5.5.1 Feedback Control

A stepper motor system is chosen based on having a confidence that the travel amount and speed are instructed by pulses, the appropriate current is supplied to the phases needed, and the motor is operated as instructed.

This system does not have a function to monitor the operating state. Therefore, if an overload state is generated or the acceleration/deceleration is too fast, the system cannot maintain the synchronization state, causing a position displacement to occur. This phenomenon is called asynchronism. As a common countermeasure against asynchronism, a margin to the torque that the motor can generate, with respect to the required torque, is given to ensure reliability. This margin is called the safety factor, and is usually about 1.5 to 2.

On the other hand, in applications where reliability is especially emphasized, an encoder can be added to a stepper motor to track its position, or a servo motor is used. A servo motor has advantages of high speed, high efficiency, and quiet noise. A servo control system, however, has drawbacks such as a complex and expensive system and poor synchronization with commands due to position deviation. Another drawback is that the gain setting requires continuous adjustment with load fluctuations.

In order to solve the problems of the conventional system, a position control system that combines a stepper motor and a position detector has been developed. However, most of them adopt a feedback control system similar to the servo motor, and include the same issues as those of the servo system.

Our  $\alpha_{\text{step}}$  product group also performs control using a position sensor, but the use of position information is significantly different from the conventional styles. This section describes the control method and advantages adopted in the  $\alpha_{\text{step}}$ .

 $\alpha_{srep}$  is the name of stepper motor products using our closed loop control system. Fig. 5.28 shows an operation concept of the  $\alpha_{srep}$ , in which the  $\theta$ -T characteristics of the  $\alpha_{srep}$  and the conventional stepper motor are superimposed in a state where the position command is fixed. The horizontal axis represents the difference between the command position and the actual position of the rotor, and the vertical axis represents the generated torque. The number of small teeth is assumed to be 50.

When torque is applied to the motor, the position shifts to a point where the generated torque is balanced with the applied torque. In the region not exceeding  $\pm 1.8^{\circ}$  against the stable point, the relationship between the applied torque and the displacement is monotonic increase, and the position is extremely stable. Therefore, no active control is performed.

Increasing the applied torque further will exceed the torque that the motor can generate, causing the rotor to rotate. The generated torque when the control is not applied is a sine wave shape, and there are a number of stable points. If the rotor rotated exceeding  $\pm 3.6^{\circ}$ , it cannot return to the original stable point even after the load torque is removed.

In the control of the *Qstep*, when the rotor is rotated by 1.8° or more, the excitation position is moved so that the maximum torque is generated at that point. In the example of Fig. 5.28, the operating point is at 4°. In this position, the generated torque in the original  $\theta$ –T characteristics shown in blue is extremely small, so the  $\theta$ -T characteristics is moved by 2.2° as shown in green to maximize the generated torque at the operating point. Performing the operation to move the  $\theta$ -T characteristics continuously while following the operating point can obtain the characteristics shown in red in the figure. There is only one stable point, so the rotor will return to the original stable point if load torque is removed. The movement is the same even during rotation. It can be seen that asynchronism does not occur if the whole figure moves at a constant speed while the relative relationship is maintained.





Fig. 5.29 is a block diagram of the  $\alpha_{\text{STEP}}$ . A high resolution resolver or an encoder is generally adopted for position detection. The deviation counter monitors the difference between the command position and the actual position, and when the difference exceeds 1.8°, a self correction operation is quickly performed on the fly to bring the difference back to 1.8°.

The current control block controls only the magnitude of the current, which is the same as a normal driver.



Fig. 5.29 Control Block Diagram of **Øster** 

### 5.5.2 Drive Based on Sine Wave

A group of alternating current motors<sup>\*1</sup>, typically induction motors, is designed for use at a constant speed, and is required to continue to generate a constant torque. In order to keep the generated torque constant, two methods are available. One is a method to design so that torque fluctuations do not occur, and the other is a method for constantly monitoring and controlling the generated torque. The latter is difficult to achieve even at present when measurement and control technologies have been evolved. Therefore, the preference has been designing to avoid torque fluctuations. When a commercial sinusoidal voltage is connected, factors other than the fundamental wave in the back EMF causes periodic fluctuations in torque, resulting in vibration and noise. In these motors, efforts have been made to exclude as much the higher order components of back EMF as possible.

On the other hand, stepper motor and brushless DC motor operations are based on "switching" the current flow to the windings, and are operated with the square wave when seeing as the current waveform. In this case, it cannot be expected that the generated torque is constant. The stepper motor has speed bands in which vibration due to torque fluctuations is large, and it is necessary to avoid these regions. In the brushless DC motor, speed fluctuations do not cause a problem at a high speed, and a speed limit is set on the low side to avoid torque fluctuations.

In the section describing the 2-phase microstep drive, we explained that the current waveform asymptotically approaches a sine wave as the step angle is divided finer. A method of "subdividing a sequence" was adopted in traditional microstep drivers, but a method of "selecting continuous sinusoidal points" is now used in recent microstep drivers. In Fig. 5.30 and Fig. 5.31, current waveforms that 360° is divided by four, eight, and sixteen are superimposed on the sine waves.



Fig. 5.30 Current Based on Sine Wave (2-phase)

Fig. 5.31 Current Trajectory Based on Sine Wave (2-phase)

Fig. 5.30 and Fig. 5.31 are the same as Fig. 5.25 and Fig. 5.26 described in the section of the 2-phase microstep drive. However, the technical details are quite different depending on whether the excitation sequence is subdivided or the points on the sine wave are selected as necessary.

Next, a 5-phase drive is described. Figure 5.32 shows the current waveform of the full step drive configured based on a sine wave. The basic sine wave is represented by a thin line. It can be seen that the actual current is generated by maintaining the value of a point obtained by dividing the basic sine wave into ten equal parts in the angular direction for a certain angle. Therefore, the current in the excitation sequence generated on switching the current is a square wave, whereas it was a sine wave. Fig. 5.33 shows the trajectory of the torque vector that combines the current for five phases. It can be seen that the electrical angle is divided by 10 equal parts.

<sup>\*1</sup> Motors that generate torque using electromagnetic induction are called induction motors. In Oriental Motor, product lines of induction motors, reversible motors, and torque motors correspond to this category.



(5-phase Full Step)

The current when the step angle is divided by 10 is shown in Fig. 5.34, and the current vector trajectory is shown in Fig. 5.35. They are almost sine waves. This allows smooth rotation even at very low speeds.



Fig. 5.34 Current Based on Sine Wave (5-phase 1/10 Step)



Fig. 5.35 Current Trajectory Based on Sine Wave (5-phase 1/10 Step)

5 Drive Circuit

# **6** Types and Features of Controllers

To control movement of a stepper motor, a controller or pulse generator that generates pulses of a set amount and frequency is required. This chapter describes types and features of controllers in addition to types of movements of stepper motors. It also describes products compatible with industrial networks essential to the system control of semiconductor manufacturing equipment and various factory production lines.

# 6.1 Operation Methods of Stepper Motors

A series of pulses generated continuously is called a "pulse train." Movement of a stepper motor can be determined by an amount (number of pulses) and speed (frequency) of the pulse train input to the driver. It can follow any pulse train within a range of rotation speed - torque characteristics, and the pulse train can freely be determined as desired.

However, if a degree of freedom is too large, it will be difficult to determine the movement of a stepper motor. Therefore, the following five types of movements are listed as typical operations. Combining these operations can achieve a series of motor movements.

- Positioning Operation<sup>\*1</sup>
- Continuous Operation
- Multi-Speed Operation
- Return-to-Mechanical Home Operation
- JOG Operation

This section describes these operations.

### 6.1.1 Positioning Operation

The most basic feature of a stepper motor is that it is possible to stop at a specific position accurately without using a sensor. This feature is used in positioning operation to rotate from the present position to the target position.

To perform positioning operation, an amount (coordinate) representing the position is required. A position on physical coordinates in an actual mechanism are represented by a rotation angle or a travel amount on a straight line, but a position on virtual coordinates that determines the rotation of the stepper motor are represented by the number of pulses or the number of steps<sup>\*2</sup>. Assuming that an amount of movement on the physical coordinate system is L, an amount of movement on the virtual coordinate system is A, and an amount of movement per step is l, the relationship between the two can be represented as follows.

$$A = \frac{L}{l} \tag{6.1}$$

<sup>\*1</sup> It is also called index operation.

<sup>\*2</sup> For drivers with built-in controller functions, since it cannot be said that a pulse train is input, the number of steps is also described. Both values are equal.

As shown in Fig. 6.1, when positioning operation is performed, the starting pulse speed (VS), the stopping pulse speed (VS), the acceleration rate (TR), the deceleration rate (TR), the operating pulse speed (VR), and the number of operating pulses (A) are set in advance, and the operation is started by a START input signal. Out of these items, the starting pulse speed and the stopping pulse speed often use the same value, and the acceleration rate and the deceleration rate also use the same value. In this example, VR and TR are applied respectively.



To determine appropriate values for the starting/stopping pulse speed and the acceleration/ deceleration rate, since torque calculation is required, details are not described here.

To specify the coordinate of the target position, there are relative positioning represented by a difference from the present position and absolute positioning specified by the unique position coordinate. Operations corresponding to these positioning methods are relative positioning operation and absolute positioning operation, respectively. Both positioning operations use numeric values on the virtual coordinate obtained by the formula (6.1).

### a. Relative Positioning Operation (Incremental Operation)

Relative positioning operation is an operating method to specify the relative difference between the present position and the target position. The number of pulses and the operating direction from the present position are specified as the position coordinate. It is suitable for applications in which the following travel amount is determined in advance or the same amount of movement is repeated, such as constant rate/fixed quantity feeding. The operating pattern of relative positioning operation is shown in Fig. 6.2.


#### b. Absolute Positioning Operation (Absolute Operation)

Absolute positioning operation is an operating method to index by specifying the target position from the reference point (home position) using the number of pulses. The reason why it is called absolute positioning operation is that one reference point is assumed as the home position. The target position can always be specified by the number of pulses counted from this reference, so it is suitable for applications in which the positioning point is frequently changed. The operating pattern of absolute positioning operation is shown in Fig. 6.3.



Fig. 6.3 Absolute Positioning Operation

#### 6.1.2 Continuous Operation

In continuous operation, the stepper motor continues to operate until the stop signal is input without specifying the number of operating pulses. This operation is used when performing an operation test of the stepper motor or moving to an arbitrary position.

As shown in Fig. 6.4, the starting pulse speed, the operating pulse speed, the acceleration/ deceleration rate, and the rotation direction are specified.



Fig. 6.4 Continuous Operation

#### 6.1.3 Multi-Speed Operation

Multi-speed operation is a method to change the operating pulse speed successively. Speed data such as the starting pulse speed, the operating pulse speed 1, the operating pulse speed 2, the acceleration/deceleration rate, and the rotation direction are specified. The operating pulse speed can be changed only in the same rotation direction. Fig. 6.5 shows the operating pattern of multi-speed operation.



Fig. 6.5 Multi-Speed Operation

#### 6.1.4 Return-to-Mechanical Home Operation

To attain accurate positioning on equipment, it is necessary to set the reference home position correctly. This home position is a position where a HOMES (home limit sensor) is fixed on the equipment, so it is called the "mechanical home position." \*3

In return-to-mechanical home operation, the mechanical home position is detected. As shown in Fig. 6.6, return operation to the home position fixed on the equipment is performed using a HOMES (home limit sensor), +LS (positive direction limit sensor), and –LS (negative direction limit sensor).

The starting pulse speed, the operating pulse speed, and the acceleration/deceleration rate are specified in return-to-mechanical home operation. Also, a movement when the table reaches both end limits and a direction of movement when the motor starts operation are specified.

Fig. 6.6 shows an example when the following operation is performed.

- Set the initial moving direction to the positive direction.
- After the table moves and inverts at +LS, it passes through the HOMES and inverts.
- Thereafter, it moves in the positive direction, and a position at which the HOMES is detected is set to the mechanical home position.



Fig. 6.6 Return-to-Mechanical Home Operation

\*3 There is a case that the table is pressed against the moving end and that position is set as the home position.

### 6.1.5 JOG Operation

As shown in Fig. 6.7, one pulse is output when a signal is input, and the motor rotates by one pulse. If the signal is kept on, the pulse signal is continuously output after a certain period of time (T) shown in the figure is elapsed. JOG operation is used for an operation test of the motor, wiring checks, and fine adjustment of the position.





# 6.2 Controller Types

A controller is a device to register a combination of frequency and pulse amounts, and oscillate pulses while switching them.

In the past, simple pulse generators to oscillate only one kind of pulse train were mainly used. Recently, the number of devices classified as controllers has been increased because they have many functions. Furthermore, in systems where multiple devices work together, more stepper motors are commanded and monitored from a master controller, so controllers with industrial network compatibility have also increased in popularity.

Drivers with built-in controller functions also increased in popularity. These drivers do not control stepper motors using a pulse train, but their movements and characteristics are not changed. There are two types of controllers, stored data type and stored program type.

#### 6.2.1 Stored Data Type

The stored data type controller registers positioning data in advance, and performs an operation when a start signal is received from a master controller, such as a programmable logic controller. Since the positioning data is stored on the stored data type controller, the master controller is only required to provide simple instructions to operate the motor.



Fig. 6.8 Stored Data Type Controller

### 6.2.2 Stored Program Type

The stored program type controller stores not only the positioning data, but also a series of movements (sequence) such as the order and the start timing of the positioning data. This controller can perform positioning operation by itself. Some controllers can also control an entire small-sized equipment in addition to I/O signals programming and conditional branching.



Fig. 6.9 Stored Program Type Controller

## 6.3 Network Control of Stepper Motors

Thanks to advances in computer technology and digital communication technology, office automation equipment has been remarkably networked, and factory automation equipment is also making rapid strides to be networked. Networks can be broadly divided into three hierarchies (Fig. 6.10). The first is the information system network (computer level), such as production management in the factory, the second is the controller network (controller level) connecting devices in the production line, and the third is the industrial network (device level) for controlling motors, sensors, actuators, and similar equipment.



Fig. 6.10 Hierarchy of Industrial Networks

The information-system network connects information management systems and production management systems in or between factories, and is a network used mainly between PCs (personal computers) and PLCs (programmable logic controllers). A large capacity of data such as design data, production data, and quality-control data required for production is transmitted mainly via Ethernet.

The controller network connects equipment in a production line, and is a network used mainly between PLCs. In addition to periodically communicating the required production information between equipment, messages such as equipment errors can be transmitted.

The industrial network is a network for equipment. It is used to connect devices such as motors and sensors to a PLC. High-speed data communication is required for synchronization of operations and speeding up of processes. The industrial network has the following advantages.

- 1. Wiring cost can be reduced because multiple signals can be controlled in the same line.
- 2. The wiring distance can be extended as compared with the pulse signal.
- 3. Design time for equipment can be reduced because protocols are defined for each network, and products can be easily integrated between different manufacturers.
- 4. Operating conditions of equipment, such as input/output information and alarm status, can be monitored, and maintainability is improved.

There are two types of networks in the category of the industrial network. One is the field network that can connect a large number and various types of devices, and the other is the motion network that enables high-speed communication and multi-axis synchronous operation.

The field network, such as CC-Link<sup>\*4</sup>, DeviceNet<sup>\*5</sup>, PROFIBUS, and Modbus, is a network that communicates the measurement information of temperatures and pressures and the operation information of motors between the upper device (master) and the lower devices (slaves). A driver compatible with this network has a built-in positioning function, and operates by receiving setting data of the speed, the rotation amount and an instruction for starting. Advantages of the field network include a high number of networked devices and long transmission distance.

The motion network, such as SSCNET<sup>\*6</sup> and MECHATROLINK<sup>\*7</sup>, has been developed to control servo motors. It can perform multi-axis synchronous operation by following operation commands given in a short cycle. On the other hand, the number of devices connected is low, and the transmission distance is also short.

<sup>\*4</sup> CC-Link (Control & Communication Link) is an open field network promoted by CC-Link Partner Association. CC-Link is a registered trademark or a trademark of Mitsubishi Electric Corporation.

<sup>\*5</sup> DeviceNet is an open field network promoted by ODVA (Open DeviceNet Vendor Association, Inc.). DeviceNet is a registered trademark or a trademark of ODVA (Open Device Net Vendors Association Inc.).

<sup>\*6</sup> SSCNET III /H is a motion network promoted by Mitsubishi Electric Corporation. SSCNET III/H is a registered trademark or a trademark of Mitsubishi Electric Corporation.

<sup>\*7</sup> MECHATROLINK-II and MECHATROLINK-III are motion networks promoted by MECHATROLINK Members Association. MECHATROLINK is a registered trademark or a trademark of the MECHATROLINK Members Association.

# 7 Theory of Stepper Motors

The principle of operation of all motors can be described as a conversion of electrical energy to mechanical energy. A stepper motor, which tends to be considered as a specific motor because of its structure, is also common in this respect. In this chapter, some topics are taken from physical points of view in order to understand the stepper motor further.

### 7.1 Dynamics of a Rotating Body

A motor is a device that converts electrical energy (input power) to mechanical energy (output power). Mechanical output power is represented by torque and rotation speed, so a basic knowledge of dynamics of a rotating body is required to understand the behavior of motors.

#### 7.1.1 Equation of Motion

The basic relationship between rotation and force is the law of motion relating to rotation, which represents the relationship between torque and angular acceleration. This corresponds to Newton's second law<sup>\*1</sup> relating to linear motion, and is represented by the following equation if the direction of the rotation axis is fixed.

$$T = J \frac{d^2 \theta_m}{dt^2} \tag{7.1}$$

In this equation, T is the effective torque, J is the moment of inertia of an object being acted upon, and  $\theta_m$  is the mechanical rotation angle.

Assuming that *J* is the total moment of inertia,  $k_D$  is the damping factor,  $T_L$  is the load torque, and  $T_M(t, \theta_m)$  is the generated torque by a motor, the following equation is obtained by clarifying the rotation of axis.

$$J \frac{d^2 \theta_m}{dt^2} + k_D \frac{d \theta_m}{dt} + T_L = T_M(t, \theta_m)$$
(7.2)

#### 7.1.2 Acceleration Torque

Term 1 of the equation of motion represents torque used for acceleration. Transposing other terms except term 1 leads to the equation below.

$$J \frac{d^2 \theta_m}{dt^2} = T_M(t, \theta_m) - k_D \frac{d \theta_m}{dt} - T_L \qquad (7.3)$$

Acceleration torque is obtained by subtracting load torque from the generated torque of a motor. And dividing the whole by the moment of inertia leads to the equation below.

$$\frac{d^2\theta_m}{dt^2} = \frac{T_M(t,\theta_m) - k_D \frac{d\theta_m}{dt} - T_L}{J}$$
(7.4)

It can be seen that the angular acceleration (acceleration of rotation) is inversely proportional to inertia.

<sup>\*1</sup> Newton's second law representing the relationship between force and acceleration: F = ma

#### 7.1.3 Viscosity Load

Term 2 of the equation of motion represents viscosity load. The viscosity load (also called viscous drag) is a load that increases in proportion to the angular speed  $d\theta_m/dt$ .

The term viscosity derives from motion in viscous fluid<sup>\*2</sup>, and in a narrow sense, only the resistance effected by fluid is referred to as viscous resistance. In a broader sense, when a load increases with respect to the speed, a portion in proportion to the speed may be extracted as viscosity load.

In motors, a component in proportion to the square of the speed in iron loss may be treated as viscosity load. Iron loss has a dimension of power. If it is divided by the angular speed to be a dimension of torque, this torque is in proportion to the speed. That is, the torque due to iron loss acts as viscosity load.

#### 7.1.4 Load Torque

Term 3 of the equation of motion represents load torque. As viscous torque is called viscous drag, the load torque is a portion of the torque without speed dependency in a load, and may be called non-viscous drag.

The load torque is a non-linear load because the magnitude of the load torque is constant but its sign changes depending on the rotation direction. This relationship is shown in Fig. 7.1.



Fig. 7.1 Frictional Load

#### 7.1.5 Generated Torque

This section describes a case where the generated torque of a stepper motor is applied to the equation of motion.

#### a. A State Where Excitation is Fixed

In this case, the  $\theta$ -T characteristics are the generated torque, so the equation of motion is approximated as follows.

$$J\frac{d^2\theta_m}{dt^2} + k_D\frac{d\theta_m}{dt} + T_L = -T_H \sin Z_R \theta_m \qquad (7.5)$$

\*2 Most of fluids including gases, although to varying degrees, all have viscosity.

This equation is obtained by adding load torque to the equation representing the single step response, and if frictional load torque is applied, a position at which the motor finally stops will change. The Fig. 7.2 compares characteristics between the motor with no frictional load and that with a load of 10% of the holding torque.



Fig. 7.2 Single Step Response for Motor with Frictional Load

#### b. During Rotation

Even when a motor rotates at a constant speed and torque, the torque fluctuates in a very short time. This is because the  $\theta$ -T characteristics are switched by switching the excitation.

Fig. 7.3 shows the torque fluctuation when a 2-phase stepper motor is rotated in full step. The vertical axis is normalized as the holding torque of 1.

The solid line represents the fluctuating torque, and the broken line represents the average torque. Blue, red, and green represent 0, 0.4, and 0.8 N·m in the average torque, respectively. The average torque is used when macroscopic movements are considered, but torque fluctuations must also be considered when microscopic movements in narrow areas are considered.

Fig. 7.4 shows the result resolving the fluctuating torque into components, with a fundamental period of 7.2°. If the average torque is low and a margin relative to a load is increased, it can see that torque fluctuation component becomes large enough where vibration may occur.



Fig. 7.3 Torque Jump and Average Torque during Rotation



Fig. 7.4 Torque Fluctuation Component

## 7.2 Back EMF and Torque

An explanation that the small teeth are attracted to each other makes it easy to understand the feature of a stepper motor's "step-like movement." However, even if it is suitable for an explanation in a standstill state, it is not suitable for torque generation in a continuously rotating state or an explanation of input/output power. This section describes movements by back EMF and input/output power, which also affects DC motors or induction motors in the same manner.

#### 7.2.1 Generation of Field

The magnetic flux from which torque is generated inside the motor is called "field." This section describes how the field of a stepper motor is generated.

Fig. 7.5 shows an image of magnetic flux inside the motor, and the magnet in the rotor is axially magnetized from left to right. The right portion of the rotor shown in pink is the north pole, and the magnetic flux is oriented from the rotor to the stator. The left portion shown in blue is the south pole, and the magnetic flux is oriented from the stator to the rotor.

Considering one of the main poles on the stator, since the ease of passage of the magnetic flux is different between a state where the small teeth on the rotor and stator are aligned and a state where they are offset as shown in Fig. 7.6, the magnetic flux increases or decreases as the rotor rotates.

Fig. 7.7 shows changes of the magnetic flux on the north and south pole sides of the main poles with respect to rotation of the rotor. The shape of a curve varies depending on the design of the small teeth, but it is generally sinusoidal. In this figure, the origin of the angle is the position where the magnetic flux on the north pole side is the largest, which is the position where the small teeth are aligned on the north pole side. At this time, the small teeth are completely offset on the south pole side, and the absolute value of the magnetic flux is the smallest.



Fig. 7.5 Magnetic Flux Inside the Motor



Fig. 7.6 Change of Magnetic Flux in Facing State



Fig. 7.7 Magnetic Flux of Main Pole

If the magnetic fluxes of the north pole and south pole sides represent  $\phi_N$  and  $\phi_S$  respectively, the following equation can be used.

$$\begin{cases} \phi_N = \phi_0 + \phi_1 \cos \theta_e \\ \phi_S = -\phi_0 + \phi_1 \cos \theta_e \end{cases}$$
(7.6)

(7.7)

The direction of the magnetic fluxes is one direction on both the north and south pole sides. If a value of the magnetic fluxes is represented using a sine wave, a certain value of offset must be provided.  $\varphi_0$  represents the offset of this magnetic flux.  $\varphi_1$  is the amplitude of the change. The angle is described in an electrical angle. Sum of the magnetic fluxes of the north pole and south pole sides are passed through the main pole and act with the windings<sup>\*3</sup>. This shows in the following equation.

$$\boldsymbol{\phi} = \boldsymbol{\phi}_N + \boldsymbol{\phi}_S = 2 \boldsymbol{\phi}_1 \cos\theta_e$$

This is the field of a stepper motor.

#### 7.2.2 Flux Linkage and Back EMF

The total number of windings n is the number of windings of one main pole multiplied by the number of main poles constituting one phase, and the field multiplied by the total number of windings is called the flux linkage  $\Phi$ .

$$\Phi = n\phi \tag{7.8}$$

According to Faraday's law of electromagnetic induction, when the flux linkage changes, a voltage is generated in the windings. This voltage is called the back EMF of the motor and is a significant term for understanding the movement of the motor. Assuming that the rotation speed of the motor shaft is  $\omega_m$ , the back EMF can be represented as follows.

$$e = \frac{d\Phi}{dt} = n \frac{d}{dt} \left( 2\phi_1 \cos\theta_e \right) = -2n\phi_1 \sin\theta_e \cdot \frac{d\theta_e}{dt} = -2n\phi_1 \sin\theta_e \cdot Z_R \omega_m \tag{7.9}$$

The following equation is used to explain the relationship between the rotation speed (mechanical angular speed)  $\omega_m$  of the shaft and the speed (electrical angular speed)  $\omega_e$  of the internal electrical and magnetic changes.

$$\omega_e = Z_R \,\omega_m \tag{7.10}$$

The back EMF is generated sinusoidally, and the magnitude is as follows.

$$|e| = 2n \phi_1 Z_R \omega_m \tag{7.11}$$

This is in proportion to the speed. The proportional constant is represented by the number of turns, the amplitude of the change of magnetic flux, and the number of small teeth. However, since the inner structure and the flow of magnetic flux are usually not known, the proportional constant is obtained by measuring the back EMF. This proportional constant is called the back EMF constant, and is represented by  $k_e$ . If this is used, the magnitude of the back EMF is as follows.

$$|e| = k_e \,\omega_m \tag{7.12}$$

#### 7.2.3 Conservation of Energy and Torque

A state where the motor is rotating while applying current to the windings to generate torque is considered. Assuming that the momentary value of current is *i*, the momentary value of back EMF is *e*, the momentary value of speed is  $\omega_m$ , and the momentary value of torque is  $T_m$ , the following equation is obtained from the relationship of the conservation of energy between the electrical input and the mechanical output.

$$ei = k_e \,\omega_m \, i = T \omega_m$$

$$\therefore T = k_e \, i$$
(7.13)

Torque is obtained by multiplying the back EMF constant by the current. Therefore, the back EMF constant is considered to be the most significant factor in evaluating the motor performance.

<sup>\*3</sup> The windings that act with the field and generate the back EMF is called the armature windings.

The back EMF constant is a proportional constant of speed and back EMF, and is also a proportional constant of torque and current. Accordingly, it is called the torque constant and may be represented by  $k_T$ . Whichever is represented, the magnitude is exactly the same.

 $k_e = k_T \tag{7.14}$ 

# 7.3 Description in d-q Coordinate<sup>\*4</sup> of a Stepper Motor

Instead of representing a voltage equation showing input/output power of the motor by an alternating current fixed for each phase, d-q coordinate is represented in the rotational coordinate synchronized with the rotation of the rotor. In fixed coordinate, a value represented as a sine wave alternating current can be handled as a direct current value in d-q coordinate. Therefore, it is often used for controlling a servo motor.

A transformation of coordinate system from alternating current coordinate to d-q coordinate is based on the assumption that the voltage and the current are sine waves. So, d-q coordinate was not applied to the control of a stepper motor that was based on "using by switching a current." However, as drivers based on the sine wave drive was developed, it has been enabled to apply description by d-q coordinate. This section explains the basic concept when description by d-q coordinate is used in a stepper motor.

#### 7.3.1 Voltage Equation of 2-Phase Stepper Motors

In mathematical terms, a stepper motor can be treated as a multipole synchronous motor. An equation representing input/output power for one phase can be explained with the following equation (7.15) using the voltage v, the current i, the back EMF e, the resistance R, and the phase inductance L at the terminal. The left-hand side represents the applied voltage, term 1 on the righthand side represents the voltage drop due to resistance, term 2 represents the voltage by self and mutual inductance, and term 3 represents the back EMF. The subscript k is the number of the target phase, and the subscript l is the phase number coupled by inductance.

$$\upsilon_k = Ri_k + \frac{d}{dt} \sum_l L_{l,k} \dot{i}_l + e_k \tag{7.15}$$

For 2-phase motors, the following equation is written.<sup>\*5</sup>

$$\begin{cases} v_a = Ri_a + L \frac{d}{dt} i_a + e_a \\ v_b = Ri_b + L \frac{d}{dt} i_b + e_b \end{cases}$$
(7.16)

Here, the following two assumptions are made on the inductance of a 2-phase stepper motor. These assumptions are not established strictly, but they are small and virtually no issue.

1) Inductance does not change.

2) Mutual inductance between phases is zero.

<sup>\*4</sup> d-axis (direct axis) and q-axis (quadrature axis) are the rotational coordinate synchronized with voltage and current. In this coordinate, since the electrical values are observed as constant values, they are particularly used in the field of control. It is used to easily understand the motor movement in this manual.

<sup>\*5</sup> For simplicity, the inductance L is set to a fixed value, and L is positioned before a differential by time in the equation.

A voltage equation is often represented by vectors as follows.

$$\begin{pmatrix} v_a \\ v_b \end{pmatrix} = \left( R + L \frac{d}{dt} \right) \begin{pmatrix} i_a \\ i_b \end{pmatrix} + \begin{pmatrix} e_a \\ e_b \end{pmatrix}$$
(7.17)

#### 7.3.2 Voltage Equation of 5-Phase Stepper Motor and Transformation to 2-Phase

Descriptions in d-q coordinate are often used in 3-phase motors. Also, there are many resources available for 3-phase motors, but no document on motors for other phases than 3-phase can be found. In particular, a 5-phase motor is a unique motor, so it is briefly described here.

Self-inductance  $L_{l,l}$  and the mutual inductance  $L_{l,k}$  for a 5-phase motor have the following relationship.

$$L_{l,k} = -\frac{1}{4}L$$
(7.18)

Using this relationship, a voltage equation is summarized as follows.

$$\begin{pmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \end{pmatrix} = \left( R + \frac{5L}{4} \frac{d}{dt} \right) \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \end{pmatrix} + \begin{pmatrix} e_a \\ e_b \\ e_c \\ e_d \\ e_e \end{pmatrix}$$
(7.19)

Symmetry of this equation is poor, so if the order and sign of the rows are replaced so that the 5×5 matrix is an identity matrix, the following equation is obtained. The order of the original phase was 36° phase in an electrical angle, but the order of the phase is 72° phase due to this replacement.

$$\begin{pmatrix} v_a \\ v_c \\ v_e \\ -v_b \\ -v_d \end{pmatrix} = \left( R + \frac{5L}{4} \frac{d}{dt} \right) \begin{pmatrix} i_a \\ i_c \\ i_e \\ -i_b \\ -i_d \end{pmatrix} + \begin{pmatrix} e_a \\ e_c \\ e_e \\ -e_b \\ -e_d \end{pmatrix}$$
(7.20)

The following equation is obtained by using a vector and matrix.

$$\mathbf{v}_5 = \mathbf{Z}_5 \mathbf{i}_5 + \mathbf{e}_5 \tag{7.21}$$

Z is a matrix representing impedance. The subscript indicates an equation of 5-phase system. Since the equation with five unknowns is difficult to treat, transforming to 2-phase is considered. This is a transformation from five dimensions to two dimensions. Therefore, the degree of freedom in the system decreases from 5 to 2, but it is not inconsistency given that the balanced alternative system has only two degrees of freedom, amplitude and phase.

A transformation matrix is as follows.

$$\mathbf{C}_{52} = \sqrt{\frac{2}{5}} \begin{pmatrix} \cos 0 & \cos \frac{2\pi}{5} & \cos \frac{4\pi}{5} & \cos \frac{6\pi}{5} & \cos \frac{8\pi}{5} \\ \sin 0 & \sin \frac{2\pi}{5} & \sin \frac{4\pi}{5} & \sin \frac{6\pi}{5} & \sin \frac{8\pi}{5} \end{pmatrix}$$
(7.22)

In this equation, an angle is represented as a radian. If the equation (7.21) is multiplied by this from the left, the following is obtained.

$$\mathbf{v}_2 = \mathbf{C}_{52}\mathbf{v}_5 = \mathbf{C}_{52}\mathbf{Z}_5\mathbf{C}_{52}^+ \mathbf{C}_{52}\mathbf{i}_5 + \mathbf{C}_{52}\mathbf{e}_5 = \mathbf{Z}_2\mathbf{i}_2 + \mathbf{e}_2$$
(7.23)

The relationship between electrical values in two-dimensional coordinate and that in five-dimensional coordinate is as follows.

$$\begin{cases} \mathbf{v}_{2} = \mathbf{C}_{52}\mathbf{v}_{5} \\ \mathbf{Z}_{2} = \mathbf{C}_{52}\mathbf{Z}_{5}\mathbf{C}_{52}^{+} \\ \mathbf{i}_{2} = \mathbf{C}_{52}\mathbf{i}_{5} \\ \mathbf{e}_{2} = \mathbf{C}_{52}\mathbf{e}_{5} \end{cases}$$
(7.24)

The transformation matrix  $C_{52}$  is not a square matrix because it represents a transformation that the number of dimensions is decreased from 5-parameter system to 2-parameter system. Therefore,  $C_{52}^+$ , which is multiplied from the right side of the impedance matrix, is not an inverse matrix but a pseudo inverse matrix. In the transformation used here, the following relationship is established.

$$C_{52}^{+} = {}^{t}C_{52} \tag{7.25}$$

Assuming that applied voltage, current, and back EMF are sinusoidal, performing a transformation will obtain as follows.

#### a. Back EMF

Back EMF in a 5-phase system is represented as follows.

$$\mathbf{e}_{5} = -e_{0} \begin{pmatrix} \sin\theta \\ \sin\left(\theta - \frac{2\pi}{5}\right) \\ \sin\left(\theta - \frac{4\pi}{5}\right) \\ \sin\left(\theta - \frac{6\pi}{5}\right) \\ \sin\left(\theta - \frac{8\pi}{5}\right) \end{pmatrix}$$
(7.26)

At this time, back EMF in a 2-phase system is represented as follows.

$$\mathbf{e}_2 = \mathbf{c}_{52} \mathbf{e}_5 = e_0 \sqrt{\frac{5}{2}} \begin{pmatrix} -\sin\theta\\\cos\theta \end{pmatrix}$$
(7.27)

#### b. Applied Voltage

Applied voltage in a 5-phase system is represented as follows.  $\delta$  is a variable representing a phase difference.

$$\mathbf{v}_{5} = v_{0} \begin{pmatrix} \cos(\theta + \delta) \\ \cos\left(\theta + \delta - \frac{2\pi}{5}\right) \\ \cos\left(\theta + \delta - \frac{4\pi}{5}\right) \\ \cos\left(\theta + \delta - \frac{6\pi}{5}\right) \\ \cos\left(\theta + \delta - \frac{8\pi}{5}\right) \end{pmatrix}$$
(7.28)

At this time, applied voltage in a 2-phase system is represented as follows.

$$\mathbf{v}_2 = \mathbf{c}_{52} \mathbf{v}_5 = \upsilon_0 \sqrt{\frac{5}{2}} \begin{pmatrix} \cos(\theta + \delta) \\ \sin(\theta + \delta) \end{pmatrix}$$
(7.29)

#### c. Current

Current in a 5-phase system is represented as follows.  $\gamma$  is a variable representing a phase difference.

$$\mathbf{i}_{5} = \mathbf{i}_{0} \begin{pmatrix} \cos(\theta + \gamma) \\ \cos\left(\theta + \gamma - \frac{2\pi}{5}\right) \\ \cos\left(\theta + \gamma - \frac{4\pi}{5}\right) \\ \cos\left(\theta + \gamma - \frac{6\pi}{5}\right) \\ \cos\left(\theta + \gamma - \frac{8\pi}{5}\right) \end{pmatrix}$$
(7.30)

At this time, current in a 2-phase system is represented as follows.

$$\mathbf{i}_2 = \mathbf{c}_{52} \mathbf{i}_5 = \dot{i}_0 \sqrt{\frac{5}{2}} \begin{pmatrix} \cos(\theta + \gamma) \\ \sin(\theta + \gamma) \end{pmatrix}$$
(7.31)

#### d. Impedance Matrix

Impedance matrix in a 5-phase system is represented as follows.  $I_5$  is an identity matrix of 5 rows and 5 columns.

$$\mathbf{Z}_{5} = \left(R + \frac{5L}{4}\frac{d}{dt}\right)\mathbf{I}_{5}$$
(7.32)

At this time, impedance matrix in a 2-phase system is represented as follows.

$$\mathbf{z}_{5} = \mathbf{c}_{52} \mathbf{z}_{5} \mathbf{c}_{52}^{t} = \begin{pmatrix} R + \frac{5}{4} \frac{d}{dt} L & 0 \\ 0 & R + \frac{5}{4} \frac{d}{dt} L \end{pmatrix}$$
(7.33)

To summarize these items, a voltage equation in a 2-phase system is as follows.

$$v_0 \begin{pmatrix} \cos(\theta + \delta) \\ \sin(\theta + \delta) \end{pmatrix} = \begin{pmatrix} R + \frac{5}{4} \frac{d}{dt} L & 0 \\ 0 & R + \frac{5}{4} \frac{d}{dt} L \end{pmatrix} i_0 \begin{pmatrix} \cos(\theta + \gamma) \\ \sin(\theta + \gamma) \end{pmatrix} + e_0 \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$
(7.34)

Accordingly, under an assumption that the applied voltage, the current, and the back EMF are sinusoidal respectively, the voltage equation of a 5-phase system is identical to that of a 2-phase system.

#### 7.3.3 d-q Conversion

Applied voltage, current, back EMF, and impedance matrix in a 2-phase system are represented as follows.

$$\mathbf{v}_{2} = v_{0} \begin{pmatrix} \cos(\theta + \delta) \\ \sin(\theta + \delta) \end{pmatrix}$$
$$\mathbf{i}_{2} = i_{0} \begin{pmatrix} \cos(\theta + \gamma) \\ \sin(\theta + \gamma) \end{pmatrix}$$
$$\mathbf{e}_{2} = e_{0} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$
$$\mathbf{z}_{2} = \begin{pmatrix} R + \frac{d}{dt} L & 0 \\ 0 & R + \frac{d}{dt} L \end{pmatrix}$$
(7.35)

Also, assuming that rotation speed is constant and electrical angular speed is  $\omega$ , the following equation is obtained.

$$\theta = \omega t \tag{7.36}$$

These can be summarized as follows.

$$\upsilon_0 \begin{pmatrix} \cos(\omega t + \delta) \\ \sin(\omega t + \delta) \end{pmatrix} = \begin{pmatrix} R & -L\omega \\ L\omega & R \end{pmatrix} i_0 \begin{pmatrix} \cos(\omega t + \gamma) \\ \sin(\omega t + \gamma) \end{pmatrix} + e_0 \begin{pmatrix} -\sin\omega t \\ \cos\omega t \end{pmatrix}$$
(7.37)

Taking the equation (7.38) as a transformation matrix, to rotational coordinate, the equations (7.39) are obtained.

$$\mathbf{c}_{dq} = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix}$$
(7.38)  
$$\mathbf{v}_{dq} = \mathbf{z}_{dq} \, \mathbf{i}_{dq} + \mathbf{e}_{dq}$$
$$v_0 \begin{pmatrix} \cos\delta \\ \sin\delta \end{pmatrix} = \begin{pmatrix} R & -L\omega \\ L\omega & R \end{pmatrix} \mathbf{i}_0 \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} + \mathbf{e}_0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
(7.39)

It can be found that these equations are expressed as direct current because a variable of time has been eliminated. The first line represents a d-axis component and the second line represents a q-axis component.

Comparing the equations (7.37) and (7.39), the vector length and phase for each of voltage, current, and back EMF do not change due to transformation from a 2-phase alternate current system to d-q coordinate. In other words, it is fixed to d-q coordinate while the mutual relationship is maintained. Impedance matrix is also not changed.

#### 7.3.4 Description of Torque

If the equation (7.39) is multiplied by the current vector from the left side, the following equation is obtained.

$$i_0 v_0 \cos(y - \delta) = i_0^2 R + i_0 e_0 \sin \gamma$$

The left-hand side represents electrical input power. Term 1 on the right-hand side represents a value converted to heat as Joule loss, and term 2 represents mechanical output power. Torque is the output power divided by axis speed  $\omega_m$ , and the following equation is obtained.

(7.40)

$$T = \frac{i_0 e_0 \sin \gamma}{\omega_m} = i_0 \sin \gamma \frac{e_0}{\omega_m}$$
(7.41)

Back EMF is in proportion to speed, and the proportional constant  $\frac{e_0}{\omega_m}$  is back EMF constant  $k_e$ . In addition,  $i_0 \sin \gamma$  is a q-axis component of current. If this is given by  $i_q$ , the equation (7.41) can be represented as follows.

$$T = k_e \, i_q \tag{7.42}$$

This equation represents torque in d-q coordinate.

Solving the equation (7.39) with the current as an unknown number obtains the following equation.

$$\mathbf{i}_{dq} = \mathbf{z}_{dq}^{-1} (\mathbf{v}_{dq} - \mathbf{e}_{dq})$$

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} R & -L\omega \\ L\omega & R \end{pmatrix}^{-1} \left\{ v_0 \begin{pmatrix} \cos\delta \\ \sin\delta \end{pmatrix} - e_0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

$$= \frac{1}{R^2 + L^2 \omega^2} \begin{pmatrix} Rv_0 \cos\delta + L\omega v_0 \sin\delta - L\omega e_0 \\ Rv_0 \sin\delta - L\omega v_0 \cos\delta - Re_0 \end{pmatrix}$$
(7.43)

When the following equation is applied, q-axis component of current is maximized.

$$\delta = \pi - \tan^{-1} \left( R/L\omega \right) \tag{7.44}$$

And a value of q-axis component of current is as follows.

$$\dot{i}_{qmax} = \frac{\upsilon_0 \sqrt{R^2 + L^2 \omega^2} - Re_0}{R^2 + L_0^2 \omega^2}$$
(7.45)

If this is multiplied by torque constant, the maximum torque of this speed can be obtained in a state of operating at the constant voltage drive.

At the constant current drive, the maximum torque is applied when a value represented the setting current by d-q axis becomes q-axis component of current.

#### 7.3.5 **Description of Iron Loss**

A stepper motor has a large number of small teeth and generates high torque. Therefore, it can stably rotate without using a position detector. On the other hand, a large number of poles leads to a high frequency during operation, and this causes large iron losses. In particular, when a constantcurrent driver is used, high-speed operation can be performed, and large iron losses are generated.

The iron loss occurs in the iron core. Although the iron core is not in direct contact with windings, it receives energy through changes in magnetic force due to current. Iron loss resistance is used as a method for introducing the mechanism of this portion into an electric circuit.

In general case, a voltage equation in d-q coordinate can be represented as follows.

0

$$\mathbf{v}_{dq} = \mathbf{z}_{dq} \mathbf{i}_{dq} + \mathbf{e}_{dq}$$
$$\begin{pmatrix} v_d \\ v_g \end{pmatrix} = \begin{pmatrix} R & -L\omega \\ L\omega & R \end{pmatrix} \begin{pmatrix} i_d \\ i_g \end{pmatrix} + \begin{pmatrix} 0 \\ e \end{pmatrix}$$

Fig. 7.8 shows this equation as an electric circuit. In this way, representing electrical equipment as the form of an electric circuit is called an "equivalent circuit." Although the equivalent circuit for description in d-q coordinate is shown here, it can be represented as the equivalent circuit even if 2-phase alternate current or 5-phase alternating current is used.  $V_d$  and  $v_q$  represent terminal voltage, R represents winding resistance, L represents inductance of windings, and *e* represents back EMF.

It is necessary to note that voltage drop due to inductance in the equivalent circuit is caused by q-axis component of current on the d-axis side and q-axis component of current on the q-axis side.

A resistor is used to represent a loss in the the equivalent circuit. R in the equivalent circuit shown in Fig. 7.8 represents winding resistance. To represent an iron loss, another resistor is added to be Fig. 7.9. The added resistance  $R_c$  is called the iron loss resistance, and current flowing through the resistance is  $i_c$ . (7.46)



Fig. 7.8 Equivalent Circuit of Voltage Equation



Fig. 7.9 Equivalent Circuit Added Iron Loss Resistance

Using the iron loss resistance, the following relationship is established for voltage equation and current.

$$\begin{pmatrix} v_d \\ v_q \end{pmatrix} = \begin{pmatrix} R & -\frac{R_c + R}{R_c} \, \omega L \\ \frac{R_c + R}{R_c} \, \omega L & R \end{pmatrix} \begin{pmatrix} \dot{i}_{0d} \\ \dot{i}_{0q} \end{pmatrix} + \frac{R_c + R}{R_c} \begin{pmatrix} 0 \\ e \end{pmatrix} \quad (7.47)$$

$$i = \begin{pmatrix} 1 & -\frac{\omega L}{R_c} \\ \frac{\omega L}{R_c} & 1 \end{pmatrix} i_0 + \begin{pmatrix} 0 \\ \frac{\omega \Phi}{R_c} \end{pmatrix}$$
(7.48)

An iron loss is represented as follows.

$$L_{i} = \frac{\left\{ \dot{l}_{oq}^{2}L^{2} + \left( \dot{l}_{od}L + \frac{k_{e}}{Z_{R}} \right)^{2} \right\} \omega^{2}}{R_{c}}$$
(7.49)

The iron loss resistance is not specified as a motor constant, but it can be estimated by comparing the measured value of the iron loss with the equation (7.49). The equation (7.49) shows that the iron loss is proportional to the speed squared, but if measurement is carried out, it changes in a slightly lower order with respect to the speed. Giving a speed dependence to the iron loss resistance can correct this discrepancy, and the same method can be applied to a wide speed range. For example, if the core loss is roughly proportional to the m-th power of the speed, assuming that the core loss resistance changes with 2 to the minus m-th power of the speed, a change in the total equation (7.49) can be proportional to the m-th power of the speed.

7 Theory of Stepper Motors

# **Message from Author**

Thank you for reading the Oriental Motor Technical Manual Stepper Motor Edition.

In this technical manual, we described with a focus on having you understand the stepper motor itself. We are very glad if you understand the structure and the principle of operation that produce the accurate operation, and the characteristics that differ from other motors.

In modern society, automated equipment is used all around us. And motors are utilized to cause the movements. In addition to stepper motors described in this document, various types of motors are used, including induction motors, brushless motors, servo motors, and DC motors. Viewed from the standpoint of sizes and output powers, motors are widely used ranging from vibration motors for mobile phones in small motors to power motors for automobiles and ocean vessels in large motors. Each motor supports the movements in modern society by making use of its features.

We hope this technical manual is useful to people who read it, and leads to improvement and development of society.

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